Quasi-isotropic approximation of ray theory for anisotropic media

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SUMMARY

Wave propagation in weakly anisotropic inhomogeneous media is studied by the quasi-isotropic approximation of ray theory. The approach is based on the ray-tracing and dynamic ray-tracing differential equations for an isotropic background medium. In addition, it requires the integration of a system of two complex coupled differential equations along the isotropic ray.

The interference of the qS waves is described by traveltime and polarization corrections of interacting isotropic S waves. For qP waves the approach leads to a correction of the traveltime of the P wave in the isotropic background medium.

Seismograms and particle-motion diagrams obtained from numerical computations are presented for models with different strengths of anisotropy.

The equivalence of the quasi-isotropic approximation and the quasi-shear-wave coupling theory is demonstrated. The quasi-isotropic approximation allows for a consideration of the limit from weak anisotropy to isotropy, especially in the case of qS waves, where the usual ray theory for anisotropic media fails.

Key words: anisotropy, perturbation methods, ray theory.

INTRODUCTION

The application of ray theory to the propagation of elastic waves in inhomogeneous media is well known. The theory has been applied to isotropic and anisotropic media as well (see e.g. Babich 1961; Červený 1972; Červený, Molotkov & Pšenčík 1977; Petrashen 1980; Kashtan 1984; Petrashe & Kashtan 1984; Gajewski & Pšenčík 1990).

In this paper we present an extension to the standard ray theory for isotropic media which allows for the calculation of ray theoretical seismograms for weakly anisotropic media. For this purpose the elasticity tensor is introduced as the sum of an isotropic part of zero order and a small anisotropic part of first order. This approach is well known from the theory of electromagnetic wave propagation and is called ‘quasi-isotropic approximation’ (Kravtsov 1969; Naida 1977a; Kravtsov & Orlov 1990). The approach was applied to elastic wave propagation by Naida (1977b, 1978) and Sharafutdinov (1994).

The quasi-isotropic approximation leads to the known system of ordinary differential equations for the determination of ray paths and amplitudes in isotropic media. Additionally, one obtains two complex coupled differential equations which describe the properties of the shear waves in anisotropic media. These equations are able to describe the limit from weak anisotropy to isotropy, in contrast to the dynamic ray-tracing equations of the standard ray theory for qS waves (see e.g. Gajewski & Pšenčík 1990).

The problem of weak anisotropy and the transition to isotropy has been investigated by a number of authors, e.g. Backus (1965), Naida (1977b, 1978), Jech & Pšenčík (1989), Coates & Chapman (1990), Chapman & Shearer (1989), Guest, Thomson & Kendall (1992), Thomson, Kendall & Guest (1992), Kiselev (1994) and Sharafutdinov (1994). Our investigation leads to an especially simple result, and we show how this work relates to previous results. We show the equivalence of the quasi-isotropic approximation
The equations of motion for an inhomogeneous anisotropic medium for a fixed frequency $\omega$ are given by

$$\frac{\partial}{\partial x_k} \left( c_{iklm}(x) \frac{\partial U_m}{\partial x_l} (\omega, x) \right) + \rho \omega^2 U_m(\omega, x) = 0,$$

with elasticity tensor $c_{iklm}$, density $\rho$, displacement $U_m$ and space coordinates $x_m$.

We consider an elasticity tensor of the following form:

$$c_{iklm}(x) = c^{(0)}_{iklm}(x) + \epsilon c^{(1)}_{iklm}(x) = 2\mu(\delta_{ik}\delta_{lm} + \delta_{im}\delta_{lk}) + \epsilon c^{(1)}_{iklm}(x),$$

where $\epsilon$ is a small parameter of size $\sim \frac{1}{\omega}$.

Eq. (2) represents a weakly anisotropic medium with Lamé parameters $\lambda$ and $\mu$ for an isotropic background medium, with a first-order anisotropic term.

We look for a solution of (1) in the form of the ray series

$$U_m(\omega, x) = e^{-i\tau r(x)} \sum_{k=0}^{\infty} \frac{u^{(k)}_{lm}(x)}{(i\omega)^k}.$$

We substitute the representation of the solution (4) and the specific form of the elasticity tensor (2) into eq. (1). Collecting terms of zero order $\sim 1/(i\omega)^0$ leads to

$$c^{(0)}_{iklm} p_k p_l u^{(0)}_m - \rho u^{(0)}_m = 0,$$

with $p_k = \partial \tau / \partial x_k$. Collecting terms of first order $\sim 1/(i\omega)^1$ and $\sim \epsilon$ yields

$$c^{(0)}_{iklm} p_k p_l u^{(1)}_m - \rho u^{(1)}_m = -i\epsilon c^{(0)}_{iklm} p_k p_l u^{(0)}_m + c^{(0)}_{iklm} p_k \frac{\partial u^{(0)}_m}{\partial x_l} + \frac{\partial}{\partial x_k} \left( c^{(0)}_{iklm} p_l u^{(0)}_m \right).$$

Eq. (5) is the Christoffel equation for an isotropic medium, which leads to eikonal equations and polarizations of isotropic $P$ and $S$ waves (Červený et al. 1977).

In contrast to the case of ray theory for isotropic media, an additional term $\sim \epsilon$ appears in eq. (6). The consequences will be discussed in the following.

First we consider the compressional wave. Using the following representation for the slowness vector of the $P$ wave in the isotropic background medium:

$$p = \nabla \tau_p = \frac{1}{v_p} \mathbf{n},$$

where $\mathbf{n}$ is the unit phase normal and $v_p$ is the $P$-wave velocity, we obtain from the solution of eq. (5)

$$v_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}.$$

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and
\[ u_{ij}^{(0)} = C \mathbf{n}. \]  

(9)

The amplitude \( C \) can be obtained from the transport equation,
\[ \frac{\partial}{\partial \tau} (\rho C^2 J) - i \omega c^{(1)}_{ijkl} p_i p_j n_j n_k C^2 J = 0, \]

(10)
as the condition of solvability of eq. (6). Here we have introduced the Jacobian,
\[ J = \frac{\partial (x_1, x_2, x_3)}{\partial (\tau, \alpha, \beta)}, \]

(11)
for the transformation from ray coordinates \((\tau, \alpha, \beta)\) to Cartesian coordinates \((x_1, x_2, x_3)\).

The solution of eq. (10),
\[ C(\tau) = \phi_{\rho}(\alpha, \beta) \sqrt{\rho(\tau)} J(\tau) \exp \left[ i \omega \int_0^\tau dt' \frac{1}{p} c^{(1)}_{ijkl} p_i p_j n_k \right], \]

(12)
represents the result for the amplitude of a \( P \) wave in the isotropic background medium and additionally a traveltime shift caused by the anisotropy (Sharafutdinov 1994). The traveltime correction coincides with the results of Červeny & Jech (1982) and Hanyga (1982). The arbitrary function \( \phi_{\rho} \) of the ray coordinates \( \alpha \) and \( \beta \) in eq. (12) is defined using the initial conditions at the point \( \tau = 0 \). The function \( J(\tau) \) is obtained by solving the system of dynamic ray-tracing equations for an isotropic medium.

Now we come to the more important case of shear waves, which we discuss in detail. Here we introduce the three mutually orthogonal vectors \((\mathbf{n}, \mathbf{e}, \mathbf{h})\) forming a vector basis at every point of the ray in the isotropic background medium. \( \mathbf{n} \) is normal to the wave front and \( \mathbf{e} \) and \( \mathbf{h} \) are tangential to the wave front. \( \mathbf{n} \) is easily determined from the ray-tracing equations and \( \mathbf{e} \) and \( \mathbf{h} \) satisfy the following differential equations (Popov & Pšencík 1978; Petrashen & Kashtan 1984):
\[ \frac{\partial \mathbf{e}}{\partial \tau} = (\mathbf{e}, \nabla \mathbf{e}) \mathbf{n}, \quad \frac{\partial \mathbf{h}}{\partial \tau} = (\mathbf{h}, \nabla \mathbf{e}) \mathbf{n}. \]

(13)

Using the following representation of \( p \):
\[ p = \nabla \tau = \frac{1}{v_s} \mathbf{n}, \]

(14)
where \( v_s \) is the \( S \)-wave velocity, we obtain from the solution of eq. (5)
\[ v_s = \sqrt{\frac{\mu}{\rho}} \]

(15)
and
\[ u_{ij}^{(0)} = A \mathbf{e} + B \mathbf{h}, \]

(16)
with unknown amplitude factors \( A \) and \( B \). The values of \( A \) and \( B \) are determined from the system of transport equations
\[ c_{ijkl}^{(0)} p_i \frac{\partial u_{ij}^{(0)}}{\partial x_i} + \frac{\partial}{\partial x_k} \left( c_{ijkl}^{(0)} p_i u_{ij}^{(0)} \right) e_i - i \omega c_{ijkl}^{(1)} p_i p_j n_k u_{ij}^{(0)} = 0, \]

(17)
as the condition of solvability of eq. (6) with \( u^{(0)} \) defined in eq. (16).

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The transport equations (17) can be converted to the following system of ordinary differential equations along the ray:

\[ \frac{\partial}{\partial t} (pA^2 J) - i \epsilon_0 (A^2 pJS_{11} + ABpJS_{12}) = 0, \]  

\[ \frac{\partial}{\partial t} (pB^2 J) - i \epsilon_0 (ABpJS_{12} + B^2 pJS_{22}) = 0, \]  

where we introduce the notation

\[ S_{11} = \Gamma_{jk} e_j e_k, \quad S_{22} = \Gamma_{jk} h_j h_k, \quad S_{12} = \Gamma_{jk} e_j h_k, \quad \Gamma_{jk} = \frac{1}{\rho} \epsilon_{ijkl} p_i p_l. \]  

With

\[ X_1^2 = pA^2 J, \quad X_2^2 = pB^2 J, \]  

we can rewrite (18) as

\[ \frac{\partial}{\partial t} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{i}{2} \epsilon_0 \begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}. \]  

These are two coupled complex linear differential equations controlled by the three independent variables \( S_{11}, S_{22} \) and \( S_{12} \). The elastic moduli of the medium and the slowness vectors along the ray in the isotropic background medium enter into the matrix

\[ S = \begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix}. \]  

If we know the matrix \( S \) for the points on the ray, then we can solve the differential equations (21) for each frequency and determine the amplitudes \( A(\omega) \) and \( B(\omega) \) from eq. (20). We will see that the complex amplitudes include the phase shift of the \( qS \) waves and the additional rotation of their polarization vectors caused by the anisotropy of the medium. This rotation is in the plane orthogonal to the \( S \) ray in the isotropic background medium.

With the differential equation system (21) we can describe the limit from weak anisotropy to isotropy within the ray theory formalism. In this limit the system matrix approaches the zero matrix and the result approaches the solution for the isotropic background medium. This is in contrast to the dynamic ray-tracing equations for \( qS \) waves in anisotropic media, which are singular in the limit of isotropy (see e.g. Gajewski & Psencík 1990).

It is possible to reduce the system (21) to three real equations. With

\[ X_1 = a_1 e^{i\phi_1}, \quad X_2 = a_2 e^{i\phi_2}, \]  

we obtain

\[ a_1^2 + a_2^2 = D^2, \]  

where \( D \) is a constant defined by the initial conditions. We can then write

\[ X_1 = D \cos \delta \ e^{i\phi_1}, \quad X_2 = D \sin \delta \ e^{i\phi_2}, \]  

\( \delta \) being the phase shift.

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and we obtain the three differential equations

\[ \frac{\partial \delta}{\partial \tau} = \epsilon \cos \frac{1}{2} S_{12} \sin (\varphi_2 - \varphi_1), \]  
\[ \frac{\partial \varphi_1}{\partial \tau} = \epsilon \cos \frac{1}{2} [S_{11} + S_{12} \cos (\varphi_2 - \varphi_1) \tan \delta], \]  
\[ \frac{\partial \varphi_2}{\partial \tau} = \epsilon \cos \frac{1}{2} [S_{22} + S_{12} \cos (\varphi_2 - \varphi_1) \cot \delta]. \]  

Eq. (21) corresponds to the result of Sharafutdinov (1994). We demonstrate in the following the equivalence of eq. (21) to the quasi-shear-wave coupling theory of Coates & Chapman (1990).

Instead of (21) we can derive another system of differential equations with the help of the eigenvalue problem for the matrix \( S \).

The eigenvalue problem for the symmetric matrix \( S \) is given by

\[ SR = RV, \tag{27} \]

where \( R \) is the matrix of eigenvectors and \( V \) is the diagonal matrix of eigenvalues,

\[ R = \begin{pmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{pmatrix}, \quad V = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}, \tag{28} \]

\[ \cos \psi = \sqrt{\frac{1}{2} \left[ \left( 1 + \frac{S_{22} - S_{11}}{\sqrt{(S_{22} - S_{11})^2 + 4S_{12}^2}} \right) \right]}, \]

\[ \sin \psi = \text{sign}(S_{12}) \sqrt{\frac{1}{2} \left[ \left( 1 - \frac{S_{22} - S_{11}}{\sqrt{(S_{22} - S_{11})^2 + 4S_{12}^2}} \right) \right]}, \]

\[ V_1 = \frac{1}{2} (S_{11} + S_{22} - \sqrt{(S_{22} - S_{11})^2 + 4S_{12}^2}), \]

\[ V_2 = \frac{1}{2} (S_{11} + S_{22} + \sqrt{(S_{22} - S_{11})^2 + 4S_{12}^2}). \]

The columns of the matrix \( R \) provide an approximation to the \( qS \) polarization vectors in a weakly anisotropic medium (Jech & Psencík 1989; Kiselev 1994).

At the initial point \((\tau = 0)\) of the ray we choose

\[ \begin{pmatrix} X_1(0) \\ X_2(0) \end{pmatrix} = \varphi_4(\alpha, \beta) \begin{pmatrix} \cos \psi(0) \\ -\sin \psi(0) \end{pmatrix} + \begin{pmatrix} \sin \psi(0) \\ \cos \psi(0) \end{pmatrix}, \tag{29} \]

as initial conditions of the system (21). Here the function \( \varphi_4(\alpha, \beta) \) is an arbitrary function of the ray parameters \( \alpha \) and \( \beta \).

With the transformation

\[ \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = R \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}, \tag{30} \]

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we obtain from (21) the following system of differential equations:

\[
\begin{pmatrix}
\frac{\partial}{\partial t} Y_1 \\
\frac{\partial}{\partial t} Y_2
\end{pmatrix} = \begin{pmatrix}
\frac{i}{2} \omega v_1 - \frac{\partial \psi}{\partial t} \\
\frac{i}{2} \omega v_2 + \frac{\partial \psi}{\partial t}
\end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}.
\]
(31)

where

\[
\frac{\partial \psi}{\partial t} = \text{sign} \left( S_{12} \right) \frac{S_{12} \frac{\partial S_{12}}{\partial t} - S_{11} \frac{\partial S_{11}}{\partial t}}{(S_{12} - S_{11})^2 + 4S_{12}^2}.
\]
(32)

The eigenvalues \( v_1 \) and \( v_2 \), together with the angle \( \psi \), which describes the eigenvectors, can be used as independent variables. We have the following representation of the wavefield in terms of two interacting linear polarized \( S \) waves:

\[
\begin{pmatrix}
u_{\psi} \end{pmatrix} = \frac{e^{\psi}}{\sqrt{\rho}} \left[ Y_1 (\cos \psi e - \sin \psi h) + Y_2 (\sin \psi e + \cos \psi h) \right].
\]
(33)

With a change in variables

\[
\begin{pmatrix}
Y_1 \\
Y_2
\end{pmatrix} = \begin{pmatrix}
Z_1 \exp \left[ i \text{co} \left( \frac{1}{2} \int_0^\infty d \tau V_1(\tau') \right) \right] \\
Z_2 \exp \left[ i \text{co} \left( \frac{1}{2} \int_0^\infty d \tau V_2(\tau') \right) \right]
\end{pmatrix},
\]
(34)

we transform eq. (31) and obtain

\[
\begin{pmatrix}
\frac{\partial}{\partial t} Z_1 \\
\frac{\partial}{\partial t} Z_2
\end{pmatrix} = \begin{pmatrix}
0 & -\exp \left[ i \text{co} \left( \frac{1}{2} \int_0^\infty d \tau (V_2 - V_1) \right) \right] \\
\exp \left[ -i \text{co} \left( \frac{1}{2} \int_0^\infty d \tau (V_2 - V_1) \right) \right] & 0
\end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}.
\]
(35)

The integro-differential equations (36) were originally obtained by Coates & Chapman (1990) using an approach based on a scattering integral.

We note that the equivalent systems of eqs (21), (26), (31) and (35) must be evaluated in the frequency domain, followed by a Fourier transformation from the frequency to the time domain. We obtain a solution directly in the time domain for two special cases which we discuss next.

In the case of a homogeneous weakly anisotropic medium the matrix \( S \) is constant and the solution of eq. (21) is given by

\[
\begin{pmatrix}
X_1(\tau) \\
X_2(\tau)
\end{pmatrix} = \exp \left[ i \text{co} \left( \frac{1}{2} \int_0^\infty d \tau V_1(\tau') \right) \right] \begin{pmatrix}
\cos \psi(0) \\
- \sin \psi(0)
\end{pmatrix} + \exp \left[ i \text{co} \left( \frac{1}{2} \int_0^\infty d \tau V_2(\tau') \right) \right] \begin{pmatrix}
\sin \psi(0) \\
\cos \psi(0)
\end{pmatrix}.
\]
(36)

In the case of wave propagation within a symmetry plane of a weakly anisotropic medium, we have \( \frac{\partial \psi}{\partial t} = 0 \) for all \( \tau \) and the system (31) decouples. We obtain

\[
\begin{pmatrix}
X_1(\tau) \\
X_2(\tau)
\end{pmatrix} = \exp \left[ i \text{co} \left( \frac{1}{2} \int_0^\infty d \tau V_1(\tau') \right) \right] \begin{pmatrix}
\cos \psi(\tau) \\
- \sin \psi(\tau)
\end{pmatrix} + \exp \left[ i \text{co} \left( \frac{1}{2} \int_0^\infty d \tau V_2(\tau') \right) \right] \begin{pmatrix}
\sin \psi(\tau) \\
\cos \psi(\tau)
\end{pmatrix}.
\]
(37)

The amplitudes of the \( q \) waves in eqs (36) and (37) are independent of the frequency and they are equal to the amplitude of the \( S \) wave in the isotropic background medium. Eqs (36) and (37) further represent corrections to the traveltime of the \( S \) wave and approximations to the polarization vectors of the \( q \) waves in the plane orthogonal to the isotropic ray. The traveltime corrections have been obtained by Backus (1965), Jech & Plenčík (1989) and Kiselev (1994).

**EXAMPLES**

We will compare the results of numerical computations obtained by the quasi-isotropic approximation and by the usual ray theory for anisotropic media for the model of a vertically inhomogeneous weakly anisotropic full-space. The examples are restricted to the more interesting case of the \( q \) waves. We discuss three-component seismograms and particle-motion diagrams.

The effects of wave propagation throughout a medium are investigated without considering the radiation problem, i.e. $\varphi(x, \beta) \equiv 1$. The time signal has the form shown in Fig. 1, with a maximum frequency of 30 Hz.

The Fourier-transformed results of eqs (16), (20), (21) and (29) are compared with the results of a ray program for anisotropic media. A Cartesian coordinate system is used with the source position at (0, 0, 0) km and the receiver position at (0.5, 0, 0.9) km. The medium has a constant density, $\rho \equiv 1.0$ g cm$^{-3}$. For the isotropic background medium we use $v_r(x_1, x_2, 0 \text{ km}) = 2.0$ km s$^{-1}$, $v_s(x_1, x_2, 1 \text{ km}) = 2.5$ km s$^{-1}$ and linear interpolation of $v_r^2$ for depth levels between 0 and 1 km. The anisotropic perturbation part, $c^{(1)}_{ijkl}$, of the elasticity tensor is constructed using Hudson’s (1981) theory for crack media. A system of vertically aligned dry cracks is assumed at depth levels 0 and 1 km. The horizontal symmetry axis of the resulting hexagonal medium has an azimuth angle of 45$^0$ with the $x_1$-coordinate axis. For intermediate depth levels, $0 \text{ km} < x_3 < 1$ km, the elements of $c^{(1)}_{ijkl}$ are linearly interpolated. Three models with different strengths of anisotropy are considered using crack densities 0.1, 0.02 and 0.001.

In the case of the medium with the strongest anisotropy (crack density 0.1, Model 1) usual ray theory for anisotropic media shows that the $qS_1$ and $qS_2$ waves are separated in time after propagating through the medium (Fig. 2a). The quasi-isotropic approximation reconstructs the shear-wave splitting but the $qS_2$ wave arrives too early and the $qS$ waves are not completely separated. The $u_s/u_r$ particle-motion diagrams (Fig. 2b) show that in the quasi-isotropic approximation both $qS$ waves are polarized in the plane orthogonal to the $S$ ray in the isotropic medium. The $qS$ waves calculated by usual anisotropic ray theory have different ray paths and their polarization vectors are not coplanar.

Fig. 3 shows the results for a medium with crack density 0.02 (Model 2). The two $qS$ waves are separated in time but they superpose to a single event. The quasi-isotropic approximation gives here a similar result to the standard ray theory for anisotropic media. The seismograms obtained are clearly different from seismograms in the isotropic background medium.

In Fig. 4 the results for the medium with crack density 0.001 are shown (Model 3). This medium deviates only slightly from isotropy. As a consequence the two $qS$ waves arrive almost at the same time. Again the seismograms and particle-motion diagrams calculated by the quasi-isotropic approximation are close to those calculated by usual ray theory for anisotropic media, and both are similar to the results for the isotropic background medium (Figs 4a and 5). For comparison the results for the isotropic medium are calculated with the same initial conditions (eq. 29) as the results of the quasi-isotropic approximation.

CONCLUSIONS

We have presented a ray theory for the propagation of waves in weakly anisotropic media (the ‘quasi-isotropic approximation’). This theory takes into account all effects which distinguish the anisotropic from the isotropic case: the splitting of one $S$ wave into two $qS$ waves, the rotation of the $qS$ polarization vectors along the ray path caused by the anisotropy and the different amplitudes of the $qS$ waves. We demonstrated using numerical examples that for certain strengths of anisotropy the theory gives similar results to the usual ray theory for anisotropic media. The quasi-isotropic approximation can describe the limit from weak anisotropy to isotropy within the ray theory formalism.

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Figure 2. (a) Model 1, crack density 0.1. Seismograms calculated by ray theory for anisotropic media (left column) and by the quasi-isotropic approximation (right column). (b) Model 1, crack density 0.1. Particle-motion diagrams calculated by ray theory for anisotropic media (upper row) and by the quasi-isotropic approximation (lower row).
Figure 3. (a) Model 2, crack density 0.02. Left column: seismograms calculated by ray theory for anisotropic media (solid lines); the two qS waves (dotted lines) are superimposed. Right column: seismograms calculated by the quasi-isotropic approximation (solid lines) and by ray theory for isotropic media (dotted lines). (b) Model 2, crack density 0.02. Particle-motion diagrams calculated by ray theory for anisotropic media (upper row) and by the quasi-isotropic approximation (lower row).
Figure 4. (a) Model 3, crack density 0.001. Left column: seismograms calculated by ray theory for anisotropic media (solid lines); the two $qS$ waves (dotted lines) are superposed. Right column: seismograms calculated by the quasi-isotropic approximation (solid lines) and by ray theory for isotropic media (dotted lines). (b) Model 3, crack density 0.001. Particle-motion diagrams calculated by ray theory for anisotropic media (upper row) and by the quasi-isotropic approximation (lower row).
REFERENCES


Figure 5. Particle-motion diagrams for the isotropic full-space model corresponding to the seismograms in Fig. 4(a).