Flapping oscillations of the bent current sheet

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Abstract

We study the dependence of the flapping oscillations on the magnetotail current sheet bending, which is caused by the dipole tilt. Observations show that flapping waves propagate from the center of the current sheet to its flanks with a velocity one order of magnitude less than typical Alfvén speed. For our analysis we use the double gradient model (Erkaev et al., 2009) of the flapping oscillations, which predicts a small minimum of the total pressure (gas plus magnetic) across the current layer. It is the depth of the potential well in the total pressure which defines the period and the speed of the flapping waves. Using the extension of the Kan/Manankova equilibriums for the non-zero dipole tilt we investigate the depth of the potential well with respect to the current sheet bending rate. We show that with the growth of the dipole tilt angle the depth of the potential well becomes smaller, the period of the flapping oscillations increases, and oscillations become nonlinear. There exists the critical tilt angle, where the potential well disappears and flapping regime changes from oscillations to instability.

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1. Introduction

The flapping oscillations represent a particular mode of the large amplitude MHD waves in the magnetotail. They propagate across the magnetic field lines mostly from the center of the current sheet, where the source of oscillations is supposed to be, toward the current sheet flanks. These oscillations may be observed at a wide range of distances – up to tens of earth radii (R\textsubscript{e}) along the magnetotail current sheet. Their quasiperiod is of 2–10 min, amplitude is about 1–2 R\textsubscript{e} and group velocity is \textasciitilde 20–60 km s\textsuperscript{−1} (one order less than typical magnetosonic modes in this region). These waves were intensively investigated experimentally, e.g. see Zhang et al. (2002), Sergeev et al. (2003, 2004), Runov et al. (2005), Shen et al. (2003) and Sun et al. (2010).

Several models were proposed to describe the flapping oscillations, both kinetic (e.g. Daughton, 1998) and magnetohydrodynamic (MHD, e.g. Golovchanskaya and Maltsev, 2005; Erkaev et al., 2009; Korovinskiy et al., 2013). We follow the MHD approach by Erkaev et al. (2012), considering the magnetic field gradients as the main driving forces for the flapping waves. In the framework of this model, the wave frequency (or growth rate) is determined by a product of gradients of the tangential and normal magnetic field components along the normal (Z\textsubscript{GSM}) and tangential (X\textsubscript{GSM}) to the neutral sheet directions, respectively. The total pressure profile across the current sheet has a small minimum in this model, and this minimum is responsible for the appearance of the flapping oscillations.

In the previous studies the analysis was restricted to the symmetric background magnetic field configurations. The goal of this paper is to investigate the flapping oscillations for case of the asymmetric (bent) magnetotail current sheet.
Pure symmetric configuration of the Earth’s magnetosphere may be observed only for the strictly vertical dipole. In this case magnetospheric geometry is symmetric against GSM Z direction, the magnetotail current sheet is planar and lays in XY plane. However, the real Earth dipole has non-zero inclination in the most of time, which results in the current sheet bending in the near-Earth region and its shift as a whole in Z direction at larger distances (Tsyganenko, 2013).

In this paper the asymmetric (bent and shifted) current sheet is represented by the generalization of the Kan/Manankova kinetic equilibriums (Kan, 1973; Manankova et al., 2000), which were obtained as the solutions of kinetic Vlasov equation (see e.g. Yoon and Lui, 2005).

This paper is organized as follows: we present the brief description of the double-gradient model of the flapping oscillations in Section 2, Section 3 gives the extension of the Kan/Manankova equilibriums to the bent current sheet. We present the results of our analysis in Section 4, and the main points are highlighted and discussed in the last Section.

2. Flapping oscillations of the thin flux tube

Here we use the analytical model of the flapping oscillations (Erkaev et al., 2009, 2012), which is restricted by a very slow wave modes existing only in the presence of magnetic field gradients $\frac{\partial B}{\partial x}$ and $\frac{\partial B}{\partial z}$. This model provides the dispersion curves for both kink and sausage modes of the flapping waves, and these curves are in a good agreement with the experimental data (Sun et al., 2014) and numerical simulations (Konovitskii et al., 2013, 2015).

In the present study we analyze a thin magnetic flux tube approach for the kink-like flapping oscillations, which is valid in a short-wavelength approximation. The motion of the tubes is governed by two forces: magnetic tension and total pressure gradient as shown in Fig. 1. The thin flux tube is supposed not to disturb the background.

The simplest way to describe the flapping-waves is just to consider the motion of a plasma element of a flux tube at the center of the current sheet which is described by the equation (Erkaev et al., 2009)

$$\rho \frac{\partial V_z}{\partial t} = \rho \frac{\partial^2 z}{\partial t^2} - \frac{\partial \Pi(z)}{\partial z},$$

where $\Pi = p + \frac{\rho V_z^2}{2}$ is the total pressure, $\rho$ is plasma density and $V_z$ is the plasma tube velocity in Z direction. In the equilibrium state the gradient of the total pressure is balanced by the magnetic tension

$$\frac{\partial \Pi}{\partial z} = \frac{1}{4\pi} \left( B_z \frac{\partial B_x}{\partial x} \right),$$

where $B_x(t, z)$ may be extended near $z = 0$ including just first order term in $z$: $B_x = (\frac{\partial B_z}{\partial z})_{z=0} \cdot z$. Therefore in the symmetric case the Eq. (1) can be rearranged as

$$\frac{\partial^2 z}{\partial t^2} = -\frac{1}{4\pi\rho} \left( \frac{\partial B_x}{\partial z} \frac{\partial B_z}{\partial x} \right)_{z=0} \cdot z,$$

Thus the Eq. (1) describes the oscillations with the double-gradient frequency $\omega_f = \sqrt{\frac{1}{4\pi\rho} \left( \frac{\partial B_x}{\partial z} \frac{\partial B_z}{\partial x} \right)_{z=0}}$. In the more complex asymmetric case we have to solve the general Eq. (1). It has the integral of energy

$$\frac{1}{2} \rho \left( \frac{\partial \Pi}{\partial z} \right)^2 + \Pi = \frac{1}{2} \rho V_z^2 + \Pi = const,$$

which immediately leads to the solution

$$t = \int \frac{dz}{\sqrt{\frac{1}{2}(C - \Pi(z))}},$$

where $C$ is an integration constant and $\Pi(z)$ is the given distribution of the total pressure. This distribution can be obtained from the model of the asymmetric current sheet.

The Eq. (1) is identical to the equation of a particle motion in the potential well (Landau and Lifshitz, 1969). If the integration constant is a bit bigger than the potential minimum, then oscillations are linear. If constant $C$ has the value close to the maximum of the potential, then oscillations become nonlinear. Finally if it is over the potential well then periodic regime is absent, and particle has to leave the potential well region.

3. Generalization of the current sheet models to the asymmetric case

To provide the background asymmetric configurations with the bent current sheet, we extend the kinetic current sheet models by Kan and Manankova for the asymmetric case. Original Kan’s model (Kan, 1973) represents the magnetotail-like magnetic field lines configuration with the dipole located in the center and elongated magnetic
field lines at the both sides of it. Manankova et al. (2000) current sheet model is a more general one, it represents Kan’s potential complemented with the infinite chain of the interleaved magnetic islands (O-points) and constrictions (X-points). Both of these models are based on the isothermal solutions of the Grad–Shafranov equation. The detailed overview can be found in Yoon and Lui (2005) and Schindler and Birn (2002). Here we present the brief description.

The generating function of the more general Manankova model is

\[
g(\zeta) = p + \sqrt{1 + p^2} \exp\left(\frac{i\zeta - ib}{\zeta - a}\right),
\]

(6)
Fig. 3. Vector potentials (left column) and total pressure profiles across the current sheet at the distances $X = 13R_e$ (central column) and $X = 19R_e$ (right column) for the Manankova model. Plots were made for three different tilt angles ($\phi = 0^\circ$ in the top panels, $15^\circ$ in the central panels, and $30^\circ$ in the bottom panels).

Fig. 4. Illustration of the flux tube motion for zero tilt angle ($\phi = 0^\circ$) for the Kan model. When the thin flux tube with inside total pressure $C = 1.065$ comes to the region with the total pressure profile shown in the left panel, it starts to oscillate, as described in Section 2. The flux tube element velocity ($dZ/dt$) dependence on time is shown in the right panel.
where $\zeta = X + iZ$ is a complex variable. Solution of the Grad–Shafranov equation can be obtained from the generating function as follows (Manankova et al., 2000; Yoon et al., 2005)

$$\Psi = -\frac{1}{2} \log \left( \frac{4|g'|^2}{(1 + |g|^2)^2}, g' = \frac{\partial g(\zeta)}{\partial \zeta} \right),$$

(7)

$$\Psi = \ln \frac{pcos(X_*) + \sqrt{1 + p^2 \cosh(Z_*)}}{\sqrt{W}},$$

(8)

where $X_* = X - \frac{b(X - a)}{R^2}$, $Z_* = Z\left(1 + \frac{1}{R^2}\right)$, $W = \left(1 + \frac{1}{R^2}\right)^2$, $-4b\frac{Z_*^2}{R^2}$, $R^2 = (X - a)^2 + Z^2$.

This solution contains the singularities at $(X, Z) = (a, 0)$ and $(X, Z) = (a, \pm \sqrt{b})$. Kan solution can be obtained from Eqs. (6)–(8) when $p = 0$.

To generalize these solutions to the asymmetric case we convert the control parameters $a$ and $b$ to the complex plane. First, we replace $b \rightarrow be^{i\theta}$, which provides the dipole rotation. Next, from the empirical models of Tsyganenko (see e.g. Tsyganenko, 2013) we know, that the bent current sheet is shifted as whole from the symmetric position at

![Fig. 5. Illustration of the flux tube motion for tilt angle equal to 15°. Kan model. Total pressure profile is presented in the left panel. The flux tube element velocity is presented in right three panels for three values of the integrating constant: $C = 1.0565$ (oscillations) in the bottom panel, $C = 1.057$ (nonlinear oscillations) in the central panel, and $C = 1.058$ in the top panel. For the maximum value of $C$ the flux tube do not oscillates, but just reflects from the right border of the total pressure potential well and leave the region of calculations.](image_url)
Z = 0 plane. The existence of this shift can be factored in by replacing (Z) by (Z - ia) in (6).

Generating function and potential for the asymmetric Manankova model turn to be

\[ g(\zeta) = p + \sqrt{1 + p^2 \exp \left( i \zeta - \frac{ib\theta}{\zeta - ia} \right)}, \]

\[ \Psi = \ln \left( \frac{\sqrt{1 + 2p \cosh(z - a) + bX_{\phi}^2} + 2p \cos(x)e^{-by}}{\sqrt{1 + 2bX_{z0} + \left( \frac{b}{z^2 + (z - a)^2} \right)^2}} \right), \]

where \( X = \frac{\cos(x + (z - a) \sin \theta)}{z^2 + (z - a)^2}, \quad X_{\phi} = \frac{\partial g}{\partial \phi}, \quad X_{z0} = \frac{\partial \Psi}{\partial X_{z0}}. \)

This asymmetric field line configuration contains three singularities at \((X, Z) = (0, a)\) and \((X, Z) = (\pm \sqrt{b \sin(\frac{x}{z})}, \mp \sqrt{b \cos(\frac{x}{z})} + a)\). Since the argument of the cosine and sine functions is \( \frac{x}{z} \), the rotation of the magnetic configuration is controlled by the angle \( \frac{x}{z} \) rather than \( \phi \). In other words the tilt angle of the dipole is \( \frac{x}{z} \).

For the positive tilt angles (and positive value of the parameter \( \phi \)) with \( a < 0 \) the current sheet is bent and lifted up in the half-space with \( z > 0 \), and for the negative tilt angles and \( a > 0 \) the current sheet is shifted down.

As in the symmetric case, the potential for the asymmetric Kan model can be obtained from the Manankova asymmetric potential by setting \( p = 0 \).

Figs. 2 and 3 demonstrate how the total pressure across the sheet changes according to the tilt angle. We can see the dependence of the total pressure on \( Z \) for the different tilt angles at the fixed \( X \)-distances, namely \( X = 8R_e \) for the Kan model, \( X = 13R_e \) (across the \( X \)-point) and \( X = 19R_e \) (across the \( O \)-point) for the Manankova model. For the both models in symmetric case (\( \phi = 0 \)) we can see the local minimum at the center of the current sheet (excluding the magnetic islands in the Manankova model, as shown in Fig. 3). In accordance with the double gradient model (Erkaev et al., 2009) this minimum is responsible for the flapping oscillations of the current sheet, as it will be described below. With increase of tilt value, the potential well in the total pressure changes: the depth becomes smaller, and it is profile becomes asymmetric. For the large tilt angles over 30° this minimum disappears and transforms first to the “ledge”, and then to the maximum (which correspond to the instability in the double gradient model).

4. Flapping oscillations of the bent current sheet

Using the distributions \( \Pi(z) \) from the Kan (Manankova) models in the asymmetric current sheet we can solve the Eq. (5) and investigate how the flapping oscillations depend on the tilt angle.

We study the flux tube motion for the three different tilt angles (0°, 15°, 30°) for the current sheet models by Kan and Manankova. Results for both of the models are quite similar, so we present the results for only the Kan model. In the symmetric case the total pressure profile has the small local minimum in the center of the sheet, which leads to the linear oscillations, as shown in Fig. 4. In Fig. 5 we show the asymmetric configuration with the tilt angle 15° with the parameter \( a = 1.5R_e \) (this shift was chosen due to expectations from the empirical Tsyganenko models). The local minimum in the current sheet center becomes asymmetric, and the zero derivation values at the both sides of it (“borders” of the minimum) correspond to the different values of the total pressure.

For the fixed value of the parameter \( \phi \) the period of oscillations and amplitude of \( V_z \) grows for bigger \( C \) values inside the potential well, and the oscillations become more and more non-linear with the growth of \( C \) as shown in Fig. 5. If parameter \( C \) becomes bigger than left (smaller) border of the total pressure minimum, then we do not see any oscillations, tube just reflects from the right border and continue to drift in the opposite direction which corresponds to the instability (see Fig. 5, right top panel).

If we increase the tilt angle, then the depth of the total pressure potential well decreases, oscillations become non-linear, period increases. When the tilt angle reaches its

![Fig. 6. Dependence of the period \( T \) on the integration constant amplitude for the tilt angle equal 15° (left panel) and on the tilt angle (right panel). Dependence on \( \phi \) was plotted for the Manankova model.](image)
critical value of 30°, then pressure minimum changes into the flat “ledge”, and flapping regime changes to instability.

The oscillations case for the Manankova model are quite similar to the one for the Kan model. We can see the growth of the period of oscillations with decreasing of the total pressure minimum depth and appearance of the non-linear oscillations. At the same time in the Manankova current sheet we can find the regions (in the vicinity of X-point) where \( \frac{\partial \Pi}{\partial \xi} \) changes its sign and so the total pressure has maximum instead of minimum. These are the instability regions.

5. Discussion and conclusions

In the present paper we have investigated how the asymmetry of the magnetotail current sheet affects the flapping oscillations. For this purpose we describe the flapping oscillations in terms of the thin flux tube oscillations and present the generalization of the Kan and Manankova current sheet models for the asymmetric case.

In this case the total pressure profile has the local minimum at the center of the current sheet, unlike boundary layer approximation, commonly used in physics of the magnetopause (Shindler, 1972), where \( \Pi = const \) across the structure. The depth of the total pressure minimum is small compared to the pressure value (about two order less) and proportional to the double gradient frequency \( \frac{\partial \Pi}{\partial \xi}, \frac{\partial ^2 \Pi}{\partial \xi^2} \). One can conclude that the smallness of the potential well automatically means that produced effect will be insignificant. Nevertheless the present study shows, that the smaller value of the potential well leads to the larger period and lower wave velocity, while the amplitude may be rather large.

With the growth of the configuration asymmetry the depth of the total pressure potential well decreases, the double gradient frequency decreases and the period of the oscillations grows, as shown in Fig. 6 (right panel). At the same time the potential well becomes asymmetric, so oscillations turn to non-linear regime. The potential well becomes asymmetric with the growth of the parameter \( \phi \). Thus when we increase the amplitude (integration constant \( C \)), we first pass the lower border (the left one in Fig. 5). Then the instability appears and magnetic flux tube will move toward the smaller border. Disturbance will grow, and automatically it will be of the kink mode (asymmetric by z), thus the current sheet asymmetry may be treated as one of mechanisms for the kink mode generation.

By the other hand, our model is non-linear, so we can obtain the dependence of the oscillations period on the amplitude (the integration constant \( C \)) in the asymmetric case as shown in Fig. 6 (left panel). This period tends to infinity at the limit \( C_{LM} \) defined by the left border of the total pressure minimum. For the \( C \) value bigger than \( C_{LM} \) regime of the thin flux tube motion changes, and oscillations disappear. For the critical values of the tilt angle (strong asymmetry) the depth of the potential well goes to zero and beyond the critical angle it transforms to the maximum. Consequently, the period of the oscillations tends to infinity, and in the extreme case we suppose to see the instability instead of the oscillations.

The presented investigation was made in order to analyze the asymmetric configuration stability. We used the simple scheme with the thin flux tube oscillations, to show that the changes in the current sheet symmetry can strongly affect the processes in the magnetotail. The presented approach of the asymmetric magnetotail could be used in the future investigations of the current sheet stability.

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