THE INTERNAL MAGNETIC FIELD OF THE SUN AND
PECULIARITIES OF THE SOLAR ACTIVITY CYCLES

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(Received 9 January; in final form 20 September, 1984)

Abstract. The existence of prolonged periods of abnormally low solar activity (such as the Maunder minimum) is explained within the framework of Leighton’s model of a solar cycle with a hypothetical internal magnetic field of the Sun taken into account.

1. Introduction

Among various solar cycle models, the model by Babcock and Leighton seems to be most developed one. In this model some basic peculiarities of the temporal variation of the solar activity, such as the 11-year periodicity and the Hale and Spörer laws, are explained. At the same time, some irregularities of the cyclic variation of the solar activity, and in particular the prolonged periods of anomalously-low activity, are still far from being understood (Stix, 1981). The purpose of this paper is to try to modify the Babcock–Leighton model in such a way that it agrees with the experimental data.

As is well known, the model by Leighton (1969) supposes that the solar magnetic field is concentrated within a relatively thin surface layer where it is continuously regenerated and transformed by the differential rotation of the Sun:

$$\Omega(\text{rad yr}^{-1}) = 18 \sin^2 \theta + (\alpha + \beta \sin^n \theta) \frac{R_\odot - r}{H},$$

where \(\theta\) is the polar angle; \(\alpha, \beta,\) and \(n\) are constant parameters and \(H\) is the thickness of the convective layer.

Taking into account the equation for a frozen-in magnetic field,

$$\frac{\partial \mathbf{B}}{\partial t} = \text{rot} [\mathbf{v} \times \mathbf{B}],$$

and supposing that the magnetic tube buoys up when the magnetic field intensity exceeds a critical level \(B_c\) (of the order of some hundred gauss), Leighton obtained equations describing the evolution of the solar magnetic field components:

$$\frac{\partial B_\varphi}{\partial t} = \sin \theta \left[ (B_r + B_\varphi) \left( - \frac{R_\odot}{H} \right) (\alpha + \beta \sin^n \theta) + (36 + \frac{n \beta}{2} \sin^{n-2} \theta) B_\theta \sin \theta \cos \theta \right] - \delta \frac{B_\varphi B_\varphi}{100 B_c \tau} - \frac{B_\varphi}{50},$$

\(3a\)
\[
\frac{\partial B_r}{\partial t} = -\delta \frac{FH}{80 R_\odot \tau} \frac{\partial}{\partial \mu} (\mu B_\varphi) + \frac{1}{T_D} \frac{\partial}{\partial \mu} \left( (1 - \mu^2) \frac{\partial B_r}{\partial \mu} \right),
\]
(3b)

\[
\frac{\partial B_s}{\partial t} = -\frac{GH}{80 R_\odot \tau} \frac{\partial}{\partial \mu} (\mu B_\varphi) - \frac{B_s}{50},
\]
(3c)

\[
B_\theta = \frac{R_\odot}{H \sin \theta} \int_{-1}^{\mu} (B_r + B_s) \, d\mu,
\]
(3d)

where

\[
\delta = \begin{cases} 
0 & \text{when } |B_\varphi| \leq B_c, \\
1 & \text{when } |B_\varphi| > B_c,
\end{cases}
\]

\(\mu = \cos \theta, F \) and \( G \) are constants; \( \tau \) is the characteristic decay time of the magnetic spots (of the order of several months); \( T_D \) is the characteristic decay time of the \( B_r \) field due to turbulent diffusion (it is supposed to be approximately 22 years); and \( B_s \) is an arbitrarily-introduced radial magnetic field which, in contrast to \( B_r \), is supposed to be unaffected by the turbulent diffusion.

Under the assumption that the magnetic field at some initial moment is strictly poloidal, Leighton has numerically integrated system (3), and the results have allowed him to explain the above-mentioned regularities in the cyclic variation of solar activity.

As already noted above, a magnetic field generated by the process under consideration is supposed to concentrate within a surface layer and to be independent of any internal processes in the Sun. In such a case, the appearance of the primary poloidal magnetic field \( B_0 \) proves to be only an occasional process and, as a consequence, any interruption in the operation of the solar dynamo must result in a halt in activity, until a new poloidal magnetic field occurs on the Sun through some unknown causes.

At the same time, there is a series of models supposing the existence of an internal magnetic field on the Sun which is not associated with the processes within the convective zone (Piddington, 1972; Layzer et al., 1979). Based on these models, it seems reasonable to suppose the existence of an additional magnetic field \( B_{\text{in}} \) at the inner boundary of the convective layer caused by some internal process. As the normal component of the magnetic field is continuous, it permeates the whole convective layer; the thickness of which is supposed to be small with respect to the Sun’s radius \( R_\odot \). From the frozen-in magnetic field equation (2), one can see that the differential rotation in (1) does not affect the radial component of the field so that the variations of \( B_{r\text{in}} \) are completely determined by the variations of the internal sources. Besides, as these sources are located outside the convective zone, that field is not affected by turbulent diffusion. Thus, the field \( B_{r\text{in}} \) is, to some extent, analogous to the field \( B_r \) in Leighton’s model. This circumstance allows us to modify the Leighton model in the following manner (Pudovkin and Benevolenska, 1982):
(1) Having identified $B_s$ with $B_{r0}^n$, we shall consider it as unassociated to the processes within the convective zone and, hence, to be nondecaying.

(2) We shall define this field as some given function of variables $t, \mu$; thereby, Equation (3c) is eliminated from the system (3).

The angular dependence of $B_s(\theta, t)$ will be supposed as being in the form proposed by Leighton:

$$B_s(\theta, t) = B_{s,0}(t) \sin \theta \cos \theta.$$  \hspace{1cm} (4)

All the other parameters of the model are assumed to be the same as in the standard Leighton model: $F = 10$; $\alpha = 0$; $\beta = 10$ rad yr$^{-1}$; $n = 8$; $\tau = 0.46$ yr; $T_D = 20$ yr; $B_c = 250$ G; $H = 0.1$ $R_\odot$.

The initial conditions are also assumed to be in the standard form:

$$B_{\varphi}|_{t=0} = 0, \quad B_r|_{t=0} = 0,$$

$$B_\theta|_{t=0} = \frac{R_\odot}{H \sin \theta} \int_{-1}^{\mu} B_s|_{t=0} \, d\mu,$$

$$B_s|_{t=0} = B_{s0} \sin \theta \cos \theta.$$  \hspace{1cm} (5)

As any arbitrary function of time may be expanded into a Fourier series of elementary harmonics, we shall first consider the effect of a magnetic field varying with time as an elementary cosinusoid

$$B_{s0}(t) = B_{s0}^0 \cos \left( \frac{2\pi}{T} t \right),$$  \hspace{1cm} (6)

where $T$ is a characteristic period and large with respect to the solar cycle period; for instances, in the present calculations, $T$ was assumed to be 180 yr.

Results of the numerical integration of system (3) with initial conditions (5) are discussed below.

2. Results of the Analysis

In Figure 1a we show the temporal variation of the absolute value of the toroidal magnetic field intensity, $\langle |B_\varphi| \rangle$, averaged along the meridian, in the case $B_{s0}^0 = 1$ G. The latitudinal distribution of $B_\varphi$ is presented in Figure 1b, where the bottom panel shows the radial component variation of the internal magnetic field ($B_r$).

One can see in Figure 1 that the introduction of $B_s$ into Leighton’s model significantly changes the character of the cyclic variations of $\langle |B_\varphi| \rangle$, and what is important, due to the nonlinearity of system (3), the character of the $\langle |B_\varphi| \rangle$ variations is not only determined by the values of $B_s$ at a given moment, but also by the prehistory of the process as well. This makes the solar activity greatly sensitive to the shape of the temporal variation of $B_s(t)$. In particular, in the case where the internal field intensity’s
Fig. 1. Variations of annually-averaged magnetic field components in the case $B^0_{z,0} = 1$ G. (a) $\langle |B_\phi| \rangle$; (b) $B_\phi$; (c) $B_z$. The solid lines correspond to positive values of $B_\phi$ and $B_z$, and the dotted lines to the negative ones. The $B_\phi$ isolines correspond to the values of $\pm 100; \pm 200; \pm 400; \pm 800$ G; isolines of $B_z$ to $\pm 0.1; \pm 0.2; \pm 0.4$ G.
temporal variation shape is supposed to be sinusoidal, four significantly different regimes may be distinguished in the development of $B_\phi(t)$.

(I) Periods of regular quasi-11-year cycles (in Figure 1 these are the years from 175 to 330 and from 620 to 780). During these periods, we observe all the regularities which are typical for the variation of the solar activity, as in Leighton's standard model. At the same time, some additional features may be noticed in the development of $\langle |B_\phi| \rangle$. Indeed, one can see that the intensity of the cycles alternates in a regular manner: the cycles in which the sign of $B_\phi$ coincides with that of $B_s$ are more intensive than the cycles in which they have opposite signs. As is known, at present cycles with a southward surface field predominate (Chernosky, 1966). From this, one may conclude that the internal magnetic field (if it is responsible for the observed asymmetry of the cycles) is also directed southwards. This conclusion agrees with the results of earlier studies by Pudovkin and Benevolenska (1982) and by Levy and Boyer (1982).

(II) Periods of abnormally-low activity (in Figure 1 they correspond to the years 120–180 and 570–620), which violate the cyclic course of activity and Hale's law. To some degree, these periods may be analogous to the experimentally-found Mounder-type periods of low activity.

(III) Periods of anomalously-high activity (in Figure 1, the years 360–410). During these periods the cyclic character of the activity is also violated, although some remnants of the cyclic recurrence are still traceable; Hale's law is not fulfilled either. Whether periods of that kind have been observed in reality, is not as yet clear.

(IV) Periods of intense though greatly unstable (with respect to their duration and amplitude) oscillations of the activity (in Figure 1, the years from 0 to 170 and from 420 to 550).

It can be seen that the introduction of a hypothetical internal solar field into Leighton's model results in a great variation in the behaviour of $\langle |B_\phi| \rangle$, which depends not only on the shape of the curve of $B_{s,0}(t)$ but also on its amplitude. In Figure 2 we show the same variables as in Figure 1 but for $B_{s,0} = 0.9$ G. Comparing these two figures, one can see that even an insignificant diminution of $B_{s,0}^0$ leads to the disappearance of interruptions in the cyclic course of the solar activity. On the other hand, an increase in the value of $B_{s,0}^0$ to 1.5 G causes a complete violation of the process of Leighton's dynamo (Figure 3).

Many of the observed variations of the solar cycle parameters, therefore, may be explained not only by changes of the convective zone characteristics (such as $\Omega, \tau, T_D, B_c, \ldots$) but also by an intensity and phase variation of the internal magnetic field of the Sun. And what is important is that the existence of the internal magnetic field permits the dynamo process to recover, even after a rather prolonged interruption.

According to the present model, the state of a magnetic field on the Sun (and, hence, the solar activity level) at any given moment is closely related to its prehistory. This fact permits one to try to predict the value of $|B_\phi|_{\text{max}}$ (a measure of the solar activity) at the maximum of a given cycle from the values of the magnetic field components during the preceding years.

As was shown by Leighton (1969) and Hirshberg (1973), the radial component of the
Fig. 2. The same as in Figure 1 for $B_{x,0} = 0.9$ G.
Fig. 3. The same as in Figures 1 and 2 for the case $B^0_{\phi} = 1.5$ G.
solar magnetic field approaches its maximum in the course of the 11-year cycle 2–3 years earlier than a toroidal field. An analogous situation occurs in the Leighton’s theoretical model (in both original and modified versions) as well. In this connection, in Figure 4 we show the calculated values of $|B_\varphi|_{\text{max}}$ which depend on the maximum values of the radial component of the field $|B_r^1|_{\text{max}} = |(B_r + B_s)|_{\text{max}}$ reached in the course of the same cycles (only the first (regular) regime shown in Figure 1 is considered). As we see, the relation between the values $|B_\varphi|_{\text{max}}$ and $|B_r^1|_{\text{max}}$ is significantly close, which permits one to predict the values of $|B_\varphi|_{\text{max}}$ for approximately 2–3 years beforehand.

In the same manner, one can explain the fact that the level of solar activity at the maximum of a cycle is proportional to the intensity of the recurrent geomagnetic disturbances at the decay phase of the preceding cycle (Ohl, 1971). Indeed, the development of recurrent geomagnetic disturbances is determined by the existence of a recurrent high velocity streams in the solar wind, and these streams are known to originate from coronal holes (Krieger et al., 1973) which are characterized by a quasiradial unipolar magnetic field (Zirker, 1977). The velocity of the recurrent streams is proportional to the area of the corresponding coronal holes (Nolte et al., 1976). Having averaged the radial magnetic field over the heliographic parallel, one ascertains the mean speed of the high velocity stream to be approximately proportional to the mean radial component intensity of the global magnetic field on the Sun.

In this connection we have studied the relation between the calculated values of $|B_\varphi|_{\text{max}}$ in the course of the model cycles and the values of $|B_r^1|$ during years near the minimum of the preceding cycles. An example of such a comparison for $\cos \theta = 0.5$ is presented in Figure 5. One can see that the compared values really correlate; the correlation coefficient has a value of $r = 0.9$ which, in principle, agrees with the data by Ohl (1971).
3. Conclusion

The main results of the analysis presented above may be summarized as follows.

1. Long-term variations of solar activity, including prolonged periods of abnormally low and/or high activity, may be explained within the framework of Leighton’s model by variations of the hypothetical quasi-stationary internal magnetic field of the Sun, without any additional suppositions about the variations of the Sun’s differential rotation.

2. There exists a close relation between the values of $|B_r|_{\text{max}}$ and $|B_\varphi|_{\text{max}}$ in the course of the 11-year cycle, and this makes it possible to predict the value of $|B_\varphi|_{\text{max}}$ (and, hence, the level of solar activity) 2–3 years ahead. Besides, the intensity of the radial component of the background magnetic field of the Sun at the minimum of a cycle correlates with the intensity of $|B_\varphi|_{\text{max}}$ at the subsequent maximum of the cycle, and this may explain the regularity in the course of the solar and geomagnetic activities observed by Ohl (1971).

Thus, Leighton’s model with an internal magnetic field taken into account, explains the main regularities observed in the course of the solar activity, including the existence of prolonged periods of low activity of the Sun.

References