LOW FREQUENCY CURRENT SHEET OSCILLATIONS RELATED TO MAGNETIC FIELD GRADIENTS

N. V. Erkaev¹,², V. S. Semenov³, H. K. Biernat⁴,⁵

¹ Institute of Computational Modelling, Russian Academy of Sciences, Krasnoyarsk, Russia
² Siberian Federal University, Krasnoyarsk, 660036, Russia, e-mail: erkaev@icm.krasn.ru
³ St.Petersburg State University, Russia
⁴Space Research Institute, Austrian Academy of Sciences, Graz, Austria
⁵Institute for Theoretical Physics, University of Graz, Austria

Abstract

One fluid ideal MHD model is applied for description of current sheet flapping disturbances appearing due to a gradient of the normal magnetic field component. The wave modes are studied which are associated to the flapping waves observed in the Earth’s magnetotail current sheet. In a linear approximation, solutions are obtained for the Harris-like behavior of the background electric current density and different profiles of the plasma density across the current sheet. The current sheet can be stable or unstable in dependence on the direction of the gradient of the normal magnetic field component.

1 Introduction

Flapping oscillations of the magnetotail current sheet have been detected by many spacecraft measurements. Namely, CLUSTER observations in the Earth’s magnetotail current sheet indicated the appearance of the wave perturbations propagating along the current sheet perpendicular to the magnetic field lines. The observed cases of such waves were first described by Zhang et al. (2002). A comprehensive statistical analysis of Sergeev et al. (2003, 2004), Runov et al. (2005a, 2005b, 2006), and Petrukovich et al. (2006) has proved the existence of such kind of waves, which were identified as the “kink”-like disturbances. The plasma sheet flapping waves are interpreted as quasi-periodic dynamical structures produced by almost vertical slippage motions of the neighboring magnetic tubes (Petrukovich et al., 2006). Data analysis yields a typical frequency of the flapping waves \( \omega_f \sim 0.035 \text{s}^{-1} \) (Sergeev et al., 2003). A group speed of the flapping waves, estimated from data analysis, is in a range of a few tens (30–70) kilometers per second (Runov et al., 2005a). Spatial amplitudes and wavelengths are of the order of 2 - 5 \( R_E \) (\( R_E \) is the Earth’s radius) (Petrukovich et al., 2006). In spite of good observational background for the flapping oscillations, a physical mechanism of this phenomenon has not been understood well. Several theoretical models were introduced, but each of them has difficulty in application to this effect. In particular, the Ballooning-type mode was proposed by Golovchanskaya and Maltsev (2005). This ballooning theory implies the condition, that the wave length scale is much less than the curvature radius of the magnetic field line. This condition is more suitable for the near Earth region, but it is not fulfilled in the magnetotail plasma sheet with a small normal component of the magnetic field.

The second physical mechanism addresses to the drift kink modes investigated by Daughton (1999), Karimabadi et al. (2003) and Sitnov et al. (2004). A particular feature of these wave modes is that they can propagate only in the direction determined by the proton drift velocity.

Another theoretical model was proposed by Erkaev et al. (2007) in a framework of MHD approach. In accordance to this model, MHD flapping modes can exist due to a gradient of the normal magnetic field component along the current sheet.

In our present paper, we develop the approach of Erkaev et al. (2007, 2008). In particular, we analyze flapping wave dispersions for four different background density profiles.
2 Basic equations

We apply conventional equations of incompressible ideal magnetohydrodynamics (MHD) for nonstationary plasma sheet parameters

\[ \rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) + \nabla P = \frac{1}{4\pi} \mathbf{B} \cdot \nabla \mathbf{B}, \]
\[ \frac{\partial \mathbf{B}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{V}, \]
\[ \frac{\partial \rho}{\partial t} + \mathbf{V} \cdot \nabla \rho = 0, \quad \nabla \cdot \mathbf{V} = 0, \quad \nabla \cdot \mathbf{B} = 0. \]

Here \( \mathbf{V}, \mathbf{B}, \rho, P \) are the velocity, magnetic field, plasma density and total pressure (sum of the magnetic and plasma pressures), respectively. We consider specific wave perturbations propagating across the magnetic field lines, which are much slower than the magnetosonic modes. In this case the incompressible approximation is quite reasonable. Our study is focussed on the wave modes existing due to a gradient of the \( B_z \) component in the magnetotail current sheet along the \( x \) direction. Here the \( B_x \) component has a gradient along the \( z \) direction, and thus we consider the two magnetic gradients as key factors for the current sheet oscillations. This approach, applied by Erkaev et al. (2008) for the flapping wave oscillations, was called as “Magnetic double gradient mechanism”.

The background configuration shown in Figure 1 is considered to be rather simple with a weak dependence of the \( B_z \) component on the \( x \) coordinate

\[ \mathbf{B} = [B_x(z/\Delta), 0, B_z(x/L_x)], \quad \mathbf{V} = 0. \]

Here \( \Delta \) is a half-thickness of the current sheet, and \( L_x \) is a length scale of the \( B_z \) variation along the current sheet. We introduce two dimensionless parameters \( \epsilon = B_z(0)/B_{z\text{max}} \) and \( \nu = \Delta/L_x \), which are assumed to be small.

We consider small perturbations of the magnetic field, total pressure and velocity,

\[ \mathbf{B} = (B_x + b_x, b_y, B_z + b_z), \quad \rho = \rho_0 + \tilde{\rho}, \quad P = P_0 + p, \quad \mathbf{V} = (v_x, v_y, v_z). \]

As a first step, we make a simplifying assumption, that all wave perturbations propagating in the \( y \) direction do not depend on the \( x \) coordinate, and thus they are considered to be functions of time and two Cartesian coordinates \( (y, z) \).

Linearizing equations (1–3) for the small perturbations, we also neglect small terms \( B_z \nabla_z b_z \) and \( B_z \nabla_z b_y \) (\( \nabla_z \) is a partial derivative with respect to the axis \( z \)), and retain the main term \( b_x \nabla_x B_z \). This is justified by the condition \( B_z L_x/(B_y \Delta) \ll 1 \).
3 Linear analysis of eigenmodes

Inserting Fourier harmonics ($\propto \exp(\iota \omega t - \iota k y)$) in the linearized equations, we obtain finally a system of equations for Fourier amplitudes

\[
\iota \omega \rho_0 v_x = \frac{1}{4\pi} \left( b_z \frac{dB_x}{dz} + B_z \frac{db_z}{dz} \right),
\]

(6)

\[
\iota \omega \rho_0 v_y - \iota k p = 0, \quad \iota \omega \rho_0 v_z + \frac{dp}{dx} = \frac{1}{4\pi} b_x \frac{dB_z}{dx},
\]

(7)

\[
i \omega b_z - B_z \frac{dv_z}{dz} + v_x \frac{dB_z}{dx} = 0, \quad i \omega b_y - B_z \frac{dv_y}{dz} = 0,
\]

(8)

\[
\omega b_x + \frac{dB_z}{dz} v_z = 0,
\]

(9)

\[
\iota \omega \rho_0 v_z d\rho_0 dz - i k v_y + \frac{dv_z}{dz} = 0.
\]

(10)

Hereafter we assume that gradient $db_z/dx$ is constant, and all other quantities do not depend on the $x$ coordinate. From linearized equations (7–10), treated as a system of ordinary equations with respect to $z$, we derive a second order ordinary differential equation for the velocity perturbation

\[
\frac{1}{\rho_0} \frac{d}{dz} \left( \rho_0 \frac{dv_z}{dz} \right) + \bar{k}^2 \bar{v}_z \left( \frac{U(\bar{z})}{\bar{\omega}^2} - 1 \right) = 0,
\]

(11)

where

\[
U(z) = \frac{1}{4\pi \rho_0} \frac{\partial B_x}{\partial z} \frac{\partial B_z}{\partial x}.
\]

(12)

We consider background plasma density and magnetic field profiles given by model formulas

\[
\rho_0 = \rho^* \left( \cosh(\alpha z/\Delta) \right)^2, \quad B_x = B^* \tanh(z/\Delta).
\]

(13)

(14)

For $0 < \alpha < 1$, the current velocity determined as a ratio of the current and plasma densities has a maximum at the center of the current sheet. This maximum becomes less for larger $\alpha$. The limit $\alpha = 1$ corresponds to the case of a uniform current velocity. For uniform background plasma density ($\alpha = 0$), Equation (11) is similar to that known from the theory of tearing mode instabilities (Pritchett et al., 1991). In this case spectral problem for equation (11) has analytical solutions corresponding to “kink”-like and “sausage”-like modes. The eigenfunctions are expressed via Legendre functions ($P^\mu_\lambda$) as follows

\[
V_z = C P^\mu_\lambda(\tanh(z/\Delta)), \quad \lambda = -1/2 + [1/4 + (k\Delta \omega_f/\omega)^2]^{1/2}, \quad \mu = -k \Delta,
\]

(15)

where

\[
\omega_f = \sqrt{\frac{1}{4\pi \rho \Delta} \frac{B^*}{\Delta} \frac{\partial B_z}{\partial x} = \sqrt{\frac{1}{4\pi \rho} \left( \frac{\partial B_x}{\partial z} \right) \frac{\partial B_z}{\partial x} \left( \frac{\partial B_z}{\partial x} \right)_{z=0} \frac{\partial B_z}{\partial x}}.
\]

(16)

The eigenfrequency is proportional to $\omega_f$, which can be real or imaginary, if the product of two magnetic gradients is positive or negative. The first case corresponds to the stable flapping waves propagating to the flanks of the current sheet, and the second case is related to the unstable situation in the current sheet. Both, stable and unstable situations are illustrated in Figure 1.

For the “kink” mode, $V_z$ is an even function of the $z$ coordinate, which requires to fulfill condition $\lambda = -\mu = k \Delta$. This relation between $\lambda$ and $k$ yields equation

\[
k \Delta = -1/2 + \sqrt{1/4 + (k \Delta)^2 \omega_f^2/\omega^2}.
\]

(17)
From this equation we derive frequency as a function of wave number for the “kink” mode

\[
\omega_k = \omega_f \sqrt{\frac{k\Delta}{k\Delta + 1}}.
\]  

(18)

For the sausage mode, \(V_z\) is an odd function, which vanishes at the center of the current sheet. This mode corresponds to condition \(\lambda = k\Delta + 1\) which leads to

\[
k\Delta + 3/2 = \sqrt{1/4 + (k\Delta)^2 \omega_f^2/\omega^2}.
\]  

(19)

This equation determines the “sausage” mode frequency

\[
\omega_s = \omega_f \frac{k\Delta}{\sqrt{(k\Delta)^2 + 3k\Delta + 2}}.
\]  

(20)

In case of nonuniform background plasma density (\(\alpha \neq 0\)), we found numerical solution. The normalized frequencies \(\omega_{k,s}/\omega_f\) and wave velocities are presented in Figure 2 for \(\alpha = 0\) and \(\alpha = 0.4\). Similar plots for \(\alpha = 0.6\) and \(\alpha = 0.8\) are shown in Figure 3. This \(\alpha\) parameter determines the model profile of the background plasma density (13). One can see at the figures, that in all cases flapping wave frequency is an increasing function of the wave number, and it has saturation for \(k \to \infty\). For a fixed wave number, the frequency tends to increase to a limit value, when the \(\alpha\) parameter varies from 0 to 1. For larger \(\alpha\), behavior of the frequency as a function of wave number becomes more shallow for the most interval of \(k\), besides very small wave numbers, where it has abrupt drop to zero. In the limit case \(\alpha \to 1\), the frequency has a constant limit \(\omega \to \omega_f\) for each wave number. This is the case of uniform current velocity across the plasma sheet. The “kink” perturbations grow faster than the “sausage” ones. This instability can take place at some regions of the Earth’s magnetotail current sheet, where the \(B_z\) component decreases towards Earth.

For example, we estimate the flapping frequency for the reasonable parameters corresponding to the current sheet conditions in the Earth’s magnetotail,

\[
B_x = 20 \text{nT}, \quad B_z = 2 \text{nT}, \quad \Delta \sim R_E, \quad n_p = 0.1 \text{ cm}^{-3}, \quad k\Delta = 0.7, \quad \partial B_z/\partial x \sim B_z/L_x, \quad L_x \sim 5R_E.
\]  

(21)
Figure 3: Frequencies, group and phase velocities corresponding to $\alpha = 0.6$ (left) and $\alpha = 0.8$ (right).

Applying these parameters to Figure 2, we find the characteristic flapping frequency $\omega_f \sim 0.03$ s$^{-1}$, and also the group velocity $V_g = 60$ km/s, and phase velocity $V_{ph} = 274$ km/s.

4 Summary

In a framework of the MHD approach, flapping waves and instability are analyzed in application to the magnetotail current sheet. Important factors for our theory are the gradients of $B_x$ and $B_z$ magnetic field components along the $z$ and $x$ directions, respectively. MHD solutions are obtained for the Harris-like current density profile across the sheet and different profiles of the background plasma density. The eigenfrequency and the growth rate for the “kink” and “sausage” modes are found. For both modes, the frequencies are monotonic increasing functions of the wave number. The corresponding wave group velocities are decreasing functions of the wave number, and they vanish asymptotically for high wave numbers.

For the typical parameters of the Earth’s current sheet, the group velocity of the “kink”-like mode is estimated as a few tens of kilometers per second that is in good agreement with the CLUSTER observations. A strong decrease of the group velocity for high wave numbers means that the small scale oscillations propagate much slower than the large scale oscillations. Because of that, the propagating flapping pulse is expected to have a smooth gradual front side part, and a small scale oscillating backside part.

The double gradient flapping waves studied in our model propagate in the direction perpendicular to the planes of the background magnetic field lines, and thus they can not be stabilized by the magnetic tension. For the “kink” mode, the magnetic field planes are just shifting with respect to each other.

Acknowledgement. This work is supported by RFBR grants No 07-05-00776-a and No 07-05-00135, by Programs 2.16 and 16.3 of RAS, and by project P17100–N08 from the Austrian “Fonds zur Förderung der wissenschaftlichen Forschung”, and also by project I.2/04 from “Österreichischer Austauschdienst”.

Proceedings of the 7th International Conference "Problems of Geocosmos" (St. Petersburg, Russia, 26-30 May 2008)
References


