ON THE NATURE OF PLASMA INHOMOGENEITIES IN THE MAGNETOSPHERE

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Abstract. In this paper it is shown that the existence of spatial inhomogeneity of convection velocity and its sudden change in time can create in combine action the spatial - temporal formation which looks like inhomogeneity of plasma density (pressure), moving at convection speed (in the direction of the Earth) or at Alfvén speed towards the magnetotail. We show that the structure appears on the magnetosphere night side at the distance of 10-20 Earth radii, because of peculiarities of the electric field of convection. Plasmoids observed during geomagnetic disturbances moving towards the magnetotail, approximately at Alfvén speed, are Alfvén resonances. Even though mechanisms of generation of both convecting, and Alfvén wave perturbations are similar, conditions of excitation of the latter are harder. Therefore, not all substorms will be accompanied by generation of plasmoids. And may be intensive, but short pulses of south IMF Bz-component can generate plasmoids, but not create auroral break up of a substorm.

Introduction

The substorm “break up” nature is of prime importance for magnetospheric physics. The “break up” shows up as a sudden amplification of electron precipitations and electric currents in the night-time region of auroral oval near the Harang discontinuity. The amplification rapidly covers the whole night-time region of the oval. Akasofu [1971] affirmed that the half-sum of disturbance propagation velocity to the west and east is approximately equal to the convection velocity projection onto the ionosphere, i.e. there is a connection between the break up and the convection in the magnetosphere.

Generation of plasma inhomogeneities in the geomagnetosphere

It is known that the combined action of convection and strong pitch-angle diffusion of electrons and protons is responsible for the formation of gas pressure distribution in the magnetosphere [Kennel, 1969; Ponomarev, 1985], that is, steady bulk currents. The divergence of these bulk currents brings about a spatial distribution of field-aligned currents, i.e. magnetospheric sources of ionospheric current systems. It is known [Ponomarev, 1985] that the contents of the magnetic flux tube (MFT) to be referred to as the plasma tube (PT) throughout the text, transfers from one MFT to another in the convection process without surplus and deficiency in the case where the field lines of the magnetic flux tube are equipotential ones. This idealization is quite realistic everywhere apart from polar auroras. Then, as the PT drifting toward the Earth in a dipole field, its volume decreases in proportion to $L^4$, and the situation is the reverse for density, while pressure increases in proportion to $-L^{20/3}$. However, the process of adiabatic compression is attended by the processes of PT depletion due to pitch-angle diffusion into the loss cone. This process is described by the factor $\exp\left(-\frac{1}{\tau}\right) = \exp\left(-\frac{\int dt}{\tau}\right) = \exp\left(-\frac{\int dv}{V_t \tau}\right)$. Thus gas pressure has a maximum on each line of convection. In accordance with the equation for $p_g$ [Ponomarev, 1985], we have:

$$p_g = p_g^0 \left(\frac{L_c}{L}\right)^{20/3} \exp\left(-\frac{5}{3} \frac{1}{V_t \tau}\right)$$

Here $p_g$ is gas pressure, $L$ is the L-coordinate, $r = LR_e$ is the distance to the Earth ($R_e$ being the Earth’s radius), $V_r$ and $V_\phi$ are the radial and azimuthal components of the convection velocity of the equatorial trace of the plasma tube, respectively, and $\tau$ is the characteristic time of PT depletion due to pitch-angle diffusion(fig.1).

M.I. Pudovkin called the time of the plasma tube passage from the boundary $L_{\infty}$ to the observation point $L$ as “transport time” [Pudovkin, Semenov, 1985]. Obviously it is not longer than the period between the reversal of the Bz-component sign of the interplanetary magnetic field (IMF) and the substorm commencement.
During this time, the plasma tube covers the distance of \( L_\infty - L \) with average velocity of 
\[ \frac{(cE/4B_0)L_\infty^4}{(L_\infty - L)} \sim \frac{(cE/4B_0)L_\infty^3}{cEL_\infty^4} \]
while drifting; the time of this process is 
\[ T = \frac{4B_0R_0(L_\infty - L)^2}{cEL_\infty^4} \]
Given 
\[ T = 3000 s, \quad B_0 = 0.5, \quad R_0 = 6.37 \times 10^8 \text{ cm}, \quad E = 3 \cdot 10^{-8} \text{ CGSE}, \]
we derive \( L_\infty \) value \( \geq 10.86 \), if \( L \leq 5.43 \). Thus the plasma tube should start from the L-shell 10-12 to be found on the L-shell of the auroral oval midposition. M.I. Pudovkin considers reconnection region to be situated on the L-shell 10-12 [Pudovkin, Semenov, 1985]. There were many unsuccessful attempts to find physical processes (microscopic processes) accompanying the reconnection (i.e. existence of strong plasma turbulence implying quasi-collisional regime) in this region. What is there at these distances on the Earth night side?

The magnetic field empirical model of Mead-Fairfield [Mead, Fairfield, 1975] is based on generalization results of observations and does not contain artificial corrections of hypothetical character. Fig. 2 shows significant depression of the magnetic field on the magnetosphere night side at \( L \sim 12-15 \). All figures demonstrating the magnetic field distribution in [Mead, Fairfield, 1975] distinctly show this effect, though authors consider it to be an artifact.
There should be balance between the magnetic and gas pressure; consequently, the region of the magnetic depression should coincide with that of the increased gas pressure. Region of the gas pressure increase in one-dimensional flow under study is also the region of an increased plasma density and low convection velocity. How make plasma density (pressure) inhomogeneities move at the electric drift velocity? The main problem here is to “get” inhomogeneity into the convective stream. As pressure and density in the magnetosphere are related by the adiabatic equation, we will interpret the equation:
\[
R(x) = x_0 \left[ (x* - x) + x_0 \right] / (x* - x)^2 + x_0, \tag{2}
\]
that is an analogue of (1). Hereafter, \(x\) is the distance along the axis \(X\) in Earth radii \(R_e\), \(V\) is the convection velocity in \(R_e/s\).

Consider one-dimensional case. Let us suppose spatial density inhomogeneity: \(n'(x,t) = n_0A(t)R(x), \tag{3}\)
where \(A(t)\) is the time function, \(R(x)\) is the \(x\)-coordinate function. Let \(A(t)\) and \(R(x)\) be \(a(o)\) and \(r(k)\), respectively: \(A(t) = (1/2\pi)\int a(o) \exp(-i\omega t)\,do\) and \(R(x) = (1/2\pi)\int r(k) \exp(-ikx)\,dk\). \(\tag{4}\)
It is necessary that \(\omega = kV_0\) for frequency and spatial oscillations form the wave moving at \(V_0\) phase velocity.

Then:
\[
n'/n_0 = (x_0 / 2\pi) \int \{a(kV_0) r(k) \exp(-ik(V_0t + x))\} \,dk \tag{5}\]
Thus “resonance” adjoint oscillations with \(\omega = kV_0\) (waves moving at the convection velocity) “get” into the convection stream. The product \(a(kV_0) r(k)\) should not be small inside the integration interval (i.e., curves \(a(kV_0)\) and \(r(k)\) should coincide) for the generation process of such oscillations to be effective. According to the “resonance” \(\omega = kV_{B_0}\) spatial inhomogeneity dimension is 1; consequently, the adjoint period of temporal oscillations should be \(1/V_0\). The period of temporal oscillations is \(3 \cdot 10^5\) s (about an hour) at \(1 \sim 6 \cdot 10^9\) cm (i.e., 10 Earth radii) and the convection velocity \(2 \cdot 10^6\) cm/s. Resonance can appear at the convection velocity as well as at the Alfven velocity \(V_A\). The Earth-directed higher inhomogeneity of the magnetic field results in certain excitement conditions of the Alfven wave moving towards the Earth. These conditions are harder than those of the wave moving towards the magnetotail. Besides, the Alfven velocity is higher than the convection one; therefore frequency range is higher. If “convective waves” resonate with variations of IMF Bz-component with the period of about an hour, Alfven waves resonate with Bz-component (IMF) variations (period of 5-10 min), i.e. with steep fronts.

Plasmoids moving towards the magnetotail at the Alfven velocity during the geomagnetic disturbance are most probably Alfven resonances. Generation mechanisms of both convective and Alfven wave disturbances are similar, but excitement conditions of the latter are harder. That is why just some substorms involve plasmoid generation. Probably, these are intensive narrow (short) pulses of the south IMF Bz-component that are similar, but excitement conditions of the latter are harder. That is why just some substorms involve plasmoid generation. Probably, these are intensive narrow (short) pulses of the south IMF Bz-component that are similar, but excitement conditions of the latter are harder.

Certain form of approximating functions is an arbitrarily chosen (to make the Fourier transform operation easier). Parameters of approximating functions are deduced from generalized observation data. Recall that \(x<0\) on the night side. Position of the magnetic field depression region indicates that the inhomogeneity formation region (hereafter we will call it “wedge”) is at the distance of 10-15 \(R_e\). It means that the “tip” of this wedge is displaced at the distance of 10-12 \(R_e\) on the night side. Let us arrange that the wedge tip has formation region (hereafter we will call it “wedge”) is at the distance of 10-15 \(R_e\). It means that the “tip” of this wedge is displaced at the distance of 10-12 \(R_e\) on the night side. Let us arrange that the wedge tip has position \(x_0\). Then:

\[
A(t) = t_0 (t + t_0) / (t^2 + t_0^2), \tag{6}\]

The \(R(x)\) function describing the “bunch” of plasma density (of plasma pressure) in the region of the magnetic field decrease can also be approximated by a simple function:
\[
R(x) = x_0 \left[ (x^* - x) + x_0 \right] / (x^* - x)^2 + x_0, \tag{7}\]
Functions (6) and (7) present disturbances of a certain initial state; density variations should be considered in reference to it: \(n'_o = n_o + n'(x,t)\). Certain form of approximating functions is an arbitrarily chosen (to make the Fourier transform operation easier). Parameters of approximating functions are deduced from generalized observation data. Recall that \(x<0\) on the night side. Position of the magnetic field depression region indicates that the inhomogeneity formation region (hereafter we will call it “wedge”) is at the distance of 10-15 \(R_e\). It means that the “tip” of this wedge is displaced at the distance of 10-12 \(R_e\) on the night side. Let us arrange that the wedge tip has coordinate \(x^*\) at the instant of time \(t=0\). The \(R(x)\) function has extremum at \(x_0\) and \(x^*\):

\[
\beta(x,t) = n_o (V_0 t - x + x^*) / \left[ (V_0 t + x_0)^2 + (V_0 t - x + x^*)^2 \right], \tag{10}\]
This is a wave disturbance that results from interaction of two oscillatory motions. Thus (10) presents a dynamic wedge form (unlike the static one described by (7)). We will call the new physical object (plasma tube trace, see
We can specify the tube average velocity at the segment between the L-shell 1 and L-shell 2 for the stable electric and dipole geomagnetic field:

\[ \langle V \rangle = cE/B_0 L_{\text{eff}}^3 \]

where \( L_{\text{eff}} = L_1^2 L_2^2 / (L_1 + L_2) \)

If the segment is not long, substitution of \( L_{\text{ef}} \) for the least value from \( \{L_1, L_2\} \) will not result in mistake. Thus the convection velocity will be considered coordinate-independent in the region of the wedge formation.

Equation (10) consists of four parameters \( V_0, t_0, x_0 \) and \( x^* \). Let us determine them using problem situation. From (6) it follows that \( t_m = t_0 \), where \( t_m \) is the moment of the time-disturbance maximum. Hereafter we will consider \( t \) to be \( t = 1.8 \cdot 10^3 \) s (half of an hour), \( V_0 \) to be constant value equal to \( 1.8 \cdot 10^{-3} \) RE/s or \( 1.15 \cdot 10^6 \) cm·s\(^{-1}\). This magnitude corresponds to the electric field of \( \sim 25 \) mV/m in the ionosphere in latitude of \( \sim 68^\circ \), i.e. to a typical electric-field value in the medium-disturbed auroral ionosphere. Given transport time of 45 minutes (time between the change of the Bz-component sign and the break up), let us determine distance from \( L=5 \) to the start point, where the wedge motion of about 5 Earth radii started 40-45 minutes before. The distance \( L=5 \) corresponds to the geomagnetic latitude of \( 64^\circ \), where we can observe the extreme equatorial point of the auroral oval night side at moderate disturbances. In this case the start-point coordinate, where the plasma packet “tip” started its motion, is \( x^* = -10 \). Finally, from (8): \( x_0 = (x^* - x_0^{(2)}) / (\sqrt{2} - 1) \).

Given \( x^* = -10, x_0^{(2)} = -13, x_0 = -7.25 \).

Fig. 3 demonstrates hand-specified density distribution in the magnetic-field depression region as (7) (the static wedge). Also, Fig. 3 shows density distribution, derived from (10), for 3 sequential instants of time \( t = 0 \) s, 560 s and 1120 s. The said arguments explain formation of the space-time traveling disturbance, which results from interaction of spatial and temporal oscillations. There are many such disturbances, but significant are those with the phase velocity close to the convection one (“resonance” disturbances). If hydrodynamical flow decelerates, the plasma pressure bunch appears. In this case the only one cause of negative anomaly is that of the electric convection field.

The magnetospheric electric field depends on the solar wind parameters. The bow shock (BS) front is the main converter of the kinetic energy of the solar wind into the electromagnetic and gas-kinetic energy of the transition layer and magnetospheric processes [Ponomarev et al., 2006, (b)]. Potential of the BS front can be defined from the electric field integration at the front using continuity of a normal component of the solar wind velocity and the IMF tangential component [Ponomarev et al., 2006, (a)]:

\[ U_b = -(V_b B_0/c) y_0(b_\parallel^2 + b_\perp^2)^{1/2} \tan \phi / 2 \sin(\psi - \psi_0) \]

where \( V_b \) is the solar wind velocity, \( B_0 \) is the IMF intensity modulus, \( c \) is the velocity of light, \( b_\parallel \) and \( b_\perp \) are unit vectors of the solar-magnetospheric coordinate system, \( \phi \) is the angle between the axis \( X \) and the vector directed from the coordinate origin to this front point. The front is approximated by the paraboloid of
rotation with \( y_b \) parameter (\( y_b/2 \) is the distance from the coordinate origin to the “nose” (“head”) BS-front point). Finally, \( \tan \psi = y/z \), \( \tan \psi_0 = b_y/b_z \).

The magnetosphere is also approximated by the paraboloid of rotation with \( y_g \) parameter. Solution of the Laplace equation in parabolic coordinates yields potential \[ \text{Ponomarev et al., 2006, (a)} \]:

\[
U_g = \left[ \left( au + b/u \right) \left( cv + d/v \right) + U_0 \right] J_1(ka) I_1(kv) \sin(\psi - \psi_0) \]  \tag{11}

where \( u \) and \( v \) are parabolic coordinates, \( J_1 \) and \( I_1 \) are Bessel functions, \( u = (r + x)^{1/2} = (2r)^{1/2} \cos \phi/2 \), \( v = (r - x)^{1/2} = (2r)^{1/2} \sin \phi/2 \), \( r = (x^2 + y^2 + z^2)^{1/2} \). Hereafter distances will be measured in Earth-radius units in the Cartesian coordinate system (unless otherwise specified). The \( k \) value is chosen in such a way that the second summand of the potential (11) is nil in the magnetopause \[ \text{Ponomarev et al., 2006, (a)} \].

In this case \( k = 3.83(y_g)^{1/2} \), where \( y_g \) is the magnetosphere half-width (with the guess value of 20 Earth radii) along the Dawn-Dusk meridian. Note that the distance from the Earth to the subsolar magnetosphere point is \( y_g/2 \).

Electric potential for the magnetosphere in the XY plain at \( b_y = 0 \) (without considering the corotation field) can be written as

\[
U_g = -A(V_s B_0 z/c) y + U_0 \left[ J_1(ku) I_1(kv) \right] \]  \tag{12},

where \( A \) is a certain numerical coefficient calculated from conditions of the substance balance (substance coming through the BS front and going along the transition layer). It is the constant magnitude under steady conditions \[ \text{Ponomarev et al., 2006, (a)} \].

Y-differentiation of (12) yields y-component \( E_y^D \) of the electric field for the Dawn-Dusk meridian:

\[
E_y^D = E_{01} + E_{02} \left[ J_1(Sx)/Sx \right] \]  \tag{13}

where \( E_{01} = -AV_s B_0 z/c \), \( E_{02} = -U_0 k_2 \), \( S_x = k(2x)^{1/2} \). \( V_0 \) is velocity, \( B_z \) is a vertical component of the interplanetary magnetic field; \( U_0 \) is found using boundary conditions.

Time dependence of the electric field is expressed by time-dependent solar wind parameters \( V_0 \) and \( B_0 z \) of the component \( E_{01} \). Time dependence of the \( E_{02} \) electric field is unknown. Let us suppose that it is equal for \( E_{01} \) and \( E_{02} \). Then:

\[
E_y^D = E_{01} \left[ 1 + G J_1(1.21 \sqrt{x})/\sqrt{x} \right] \]  \tag{14}

Here \( G = E_{02}/1.21E_{01} = \text{const} \), \( E_{01} \) depends only on time (fig. 4). In the case of one-dimensional steady flow for \( y_g = 20 \):

\[
n' = \langle nV \rangle/V = n_0 V_0/V = D \left[ 1 + G J_1(1.21 \sqrt{x})/\sqrt{x} \right] \]  \tag{15}

where \( n' \) is the density disturbed value, \( n_0 \) is an undisturbed value of the plasma density, \( \langle nV \rangle \) is a time- and space-averaged value of the particle flux under undisturbed conditions, \( D \) is the function of \( t \). Equation (15) corresponds to (1) and (2) from the physical point of view. Structure of these expressions is similar to that of (1): it is a product of two functions, one of which is time-dependent, while the second is X coordinate-dependent.

![Figure 4. A profile of electric field Ey as a function of L-shell parameter: Ey(L) = 1 + J1(1.21/L)/1.21L.](image)

![Figure 5. Comparison of two curves. Solid line - accordingly to the function, describing by (7); dot line - accordingly to the expression (15).](image)
of D and G for (15) are 0.981 and 2.43, respectively. In this case both curves correlate well. The first curve is obtained on the assumption that the magnetic field depression should correspond to the density hump (with its maximum at the field minimum), and hump spatial characteristics should be connected with substorm development by delay time. The second curve is based on electric field properties resulting from the magnetosphere parabolic model. It turns out that they can be put in the same form by a simple coefficient fitting. The curve (7) maximum position (-13) should be changed (-18 RE), the G-coefficient value is 2.43.

**Conclusions**

Physical meaning of this coincidence is that the electric field theory [Ponomarev et al., 2006,(a),(b); Ponomarev, Sedykh, 2006] forecasts electric field peculiarity along the magnetotail axis with the minimum at L ~ 18, in the magnetic field depression region. This anomaly is associated with the negative anomaly of the convection velocity, positive anomalies of the density and the gas pressure. The latter should result in the magnetic field depression observed in the model MF-75 exactly in this region. The most important is that interaction between the spatial inhomogeneity of density and the temporal oscillation of the same density results in the plasma pressure inhomogeneity moving at the convection velocity towards the Earth (fig.6).

It is shown above that, according to generalized observation data, the magnetic pressure and related to it gas pressure positive anomaly on the Earth night side at L ~ 12-15 can be a source of pressure inhomogeneities moving from the magnetotail towards the Earth at the convection velocity. The convection electric field model obtained for the magnetosphere parabolic model is shown to have field depression with moderate disturbance on the magnetosphere night side at L=18. The plasma packet is a necessary and sufficient condition for the substorm “break up” appearance [Ponomarev, 1985; Ponomarev, Sedykh, 2006].

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**References**