MULTIFRACTAL AND TOPOLOGICAL ANALYSIS OF SOLAR MAGNETIC FIELD COMPLEXITY

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Abstract. The main purpose of this work is searching of the probable precursors of X-flare events using MDI data of full solar disk. To analyze this data we use the multifractal microcanonical formalism applying Choquet capacity as measure. We build the map of exponent so-called Holder exponents for every MDI fragment containing an active region. These values specify the sudden change in a picture contrast. In addition, we provide the same analyze for inverted image. After that we estimate the two first Betti numbers for both couple of MDI fragments and corresponding couple of Holder maps. The first one specify the number of image connectedness components and second one define the number of “holes” of one polarity relatively another. We suppose that the Betti numbers variation may be used as a precursor of X-flare event.

We analyze the four active regions and obtained the following result. For MDI data the Betti numbers variations of X-flare active region are very different from flare-quiet active region. Furthermore, the Betti numbers for Holder maps display the sudden change for 24 hours before the X-flare event.

Introduction

In the beginning of the work for searching the X-flare event precursors, we proceed from the following assumptions. We suppose that the X-flare event precursor (i.e. event with power more then $10^{-4} \text{Wt/m}^2$) must be connected with magnetic flux buoyancy in active region. This flux changes the topology of digital image. Consequently, the changes of this topology should be contained in variations of picture contrast gradient.

We use the Michelson-Doppler Imager (MDI) magnetograms of full solar disk\footnote{http://soi.stanford.edu/magnetic/index5.html (1024x1024 resolution, 90min discrete)}. The main difficulties we faced to challenge during working with magnetograms were connected with the features of high-resolution digital images. First of all, MDI magnetograms are characterized by high variability of contrast, what means that values of grey-level changes significantly form pixel to pixel. Second, statistics of brightness distribution of magnetograms have power law dependence as in the most nature images \cite{1,2}, so the dispersion increases without the limit with sample extension (Figure 1). That’s why, we couldn’t apply the standard methods based on second order Pearson’s statistics to magnetograms analysis.

Multifractal microcanonical formalism [3] is used as a main approach for such image processing. In addition, we use the methods of computational topology for X-flare event precursor diagnostics.

Methods

Let’s suppose that our magnetogram given as intensity field: $I(x,y); x, y \in \mathbb{Z} \times \mathbb{Z}$. Let’s define the image intensity measure by following formula [3]:

$$\mu = \lim_{\varepsilon \to 0} \left\{ \frac{1}{A} \int \left[ \frac{(I(x+\varepsilon,y)-I(x,y))^2}{\varepsilon} + \frac{(I(x,y+\varepsilon)-I(x,y))^2}{\varepsilon} \right] dS, \right\}$$

where $A$ is a compact, $dS = dx \land dy$.

If we want to use a multifractal formalism for image analysis the measure of the image should satisfy the scale invariance properties\cite{4}:

$$\mu(A) \sim r^{h(x)}.$$
In fact, the image measure yields such condition if the original image has typical spectrum (Figure 2). Apparently, such spectrum exists also for MDI magnetograms [5-7] and we could use multifractal formalism in MDI magnetogram analysis.

As a matter of fact, multifractal analysis is transition at each point of the image from measure $\mu$ to Holder exponent $\alpha$. The main idea is as follows. In neighborhood of pixel $B_1(x)$ we find the amount of grey $\mu_1$. Further, with increasing the neighborhood to $B_2(x)$ on a whole number of pixels and deriving the series of neighborhood sizes $r_1 < r_2 < r_3 < r_4 < ...$ we’ll receive the set of measures $\mu_1, \mu_2, \mu_3, \mu_4, ...$. Then in a double logarithmic scale we find Holder exponent for every pixel as the slope $\log \mu = h(x) \log r$.

However, in general the sequence of $\log \mu_1 / \log r_1; \log \mu_2 / \log r_2; \log \mu_3 / \log r_3; ...$ is not stable for MDI data. Therefore, we cannot to draw a straight line for exponent computing. To overcome this difficulty we use as a measure the Choquet capacity [8]. The main feature of such measures is monotonicity property. In other words, if $A \subseteq B$ then $\mu(A) \leq \mu(B)$. We may obtain the Choquet capacity by various methods. In figure 3, we could see the example of estimation Choquet capacities for one of neighborhood. The $\mu_{\text{max}}$ capacity is defined by maximum value in consider neighborhood. The $\mu_{\text{min}}$ capacity is defined correspondingly by

![Figure 1](image1.png)

**Figure 1**
Dispersion increases without limit during extension sample

![Figure 2](image2.png)

**Figure 2**
Obtaining multifractal spectrum for MDI magnetogram
The minimum value in consider neighborhood. The $\mu_{iso}$ capacity is defined as the number of pixels that differs from central pixel by definite level:

$$|p(i) - p(j)| \leq \delta \Rightarrow p(i) \sqcap p(j)$$

$$\mu_{iso} = \# \{ j \mid p(j) \sqcap p(i_{center}) \}.$$ 

In our work, we use the $\mu_{iso}$ capacity.

Thus for every pixel we receive correspondent Holder exponent and as a result we obtain the Holder exponent map (Figure 4). Then we invert original magnetogram and her exponent map.

Later we use the methods of computing topology [9]. We use the variation of topology invariants (calling Betti numbers) as a precursor of X-flare event. With these characteristic we distinguish one magnetogram from another. Roughly speaking, the $\beta_0$ number display the component number of image connectedness and the $\beta_1$ number display the “holes” number. For example in figure 5, it is shown the digital image piece and its Betti numbers. As we can see at this Figure there is a one connectedness area as well as the “hole” is one.

In formal approach for the Betti numbers calculation the homology theory is used [9]. Within the bounds of this theory cycles which could not be retract to point and boundaries which could be retract to point could be distinguished. Therefore, as we can see at Figure 5, the three cycles could be built. At this Figure I, J and K are cycles, but K is a boundary. I and J are homologous, but K is not. Betti numbers are defined with the set of all such cycles. $\beta_1$ gives us the number of holes and $\beta_0$ the number of connectedness.

![Figure 3](image1.png)

**Figure 3**
Corresponding Choquet capacity:
$$\mu_{max} = 255, \mu_{min} = 25, \mu_{iso} = 2(\delta = 2)$$

![Figure 4](image2.png)

**Figure 4**
The Holder exponent map for active region and background.
We compute topological invariants both for original and inverted magnetograms and for original and inverted image of Holder maps. As a result we get four couple of such numbers. So, for active region which was observed during five days we obtain time series of Betti numbers with near 80 values.

**Results**

For exploration of the original magnetogram it was decided to investigate difference of corresponding Betti numbers for positive and negative images. We found out the distinction in variation of such difference for X-flare active region and flare-quiet active region. For flare quiet regions the differences oscillates near zero, but for flare active regions Betti differences were strongly above or under the zero level (Figure 6). From physical point of view such behavior signals that during the activity in region there is a prevalence of one polarity before another.

**Figure 5**
For given image digital piece $\beta_0=1, \beta_1=1$. Cycle I and J are homologous. Cycle K retracts to point

**Figure 6**
We find out the distinction in variation of such differences for X-flare active region and flare-quiet active region. The difference oscillates near zero or in the various half planes
The second result is connected with the investigation of Holder exponent maps. It was observed that 24 hours before the X-flare event there was a sudden change in Betti numbers for Holder maps images (Figure 7). Probably this confirms our assumption about magnetic flux buoyancy before the X-flare event and changes in the image topology.

Conclusions

The results obtained during the research are very interesting. The sudden change of gradient 24 hours before the X-flare events gives us evidence that we choose the right way of investigation. Further, of course we must increase the amount of exploration regions to provide more statistical significance results. The same is correct for effects of flare active or flare quiet regions. Naturally it is very useful to distinguish only by image whether region is flare active or flare-quiet.

Reference