COMPARISON OF FLAPPING OSCILLATIONS OBSERVED BY THEMIS WITH THE DOUBLE GRADIENT MODEL

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Abstract. Flapping oscillations observed in the current sheet of the Earth’s magnetotail, represent rather slow waves propagating from the center to the flanks with a typical speed ~20-60 km/s, amplitude ~1-2Re and quasiperiod ~2-10 minutes. The relevant model is based on double gradient of magnetic field: gradient of tangential (Bₓ) component along the normal (z) direction and normal component (B_z) along the x-direction [1]. In the framework of this model the rotation of the vector of magnetic field in the plane Z-Y as well as vector of plasma velocity is investigated to find differences between kink and sausage modes of the flapping oscillations. It is also shown that the speed of the rotation of the vector (v or B) gives the fundamental parameters of the model including double gradient frequency. The theoretical results are compared to the flapping oscillations observed by space mission Themis on 03.05.2008 in the morning sector of the magnetotail. The observed rotation of the velocity vector simultaneously on two spacecrafts of Themis mission corresponds to the kink mode of the flapping oscillations. The results obtained show that data on rotation of v and B vectors can give important information about modes and characteristics of the flapping waves.

Introduction. Flapping oscillations observed in the current sheet of the Earth’s magnetotail represent vertical (Z-axis) oscillations mostly localized in YZ-plane, rather slow and propagating from the center to the flanks along Y axis with a typical speed ~20-60 km/s (for comparison, Alfven velocity ~500 km/s), amplitude ~1-2Re and quasiperiod ~2-10 minutes (Sergeev et al., 2003, 2004; Runov et al., 2005, 2006; Petrukovich et al., 2006). Theoretically there are two modes of wave motion: kink (displacement vector is the even function with respect to z) and sausage (odd). It is supposed that all observed waves do belong to kink-mode because of typical for kink mode Bₓ-component variations from negative to positive values.

Here we discuss the event of March 05, 2008, observed by Themis mission. The location of spacecraft is shown in figure 1.

Figure 1: Flapping oscillations of the current sheet observed by Themis mission at March 05, 2008 (Runov).
From figure 1, one can see that THEMIS probes P1, P2, P4, were aligned along X-axis and simultaneously observed similar variations of Bx-component (Runov, 2008). That means that we observed the same oscillations of current sheet at all points. It is proposed that observed flapping-waves are of kink mode, but the Bx-component of magnetic field stays negative during the whole period and we have no clear evidence for the wave mode. Thus, there still exists a possibility that flapping waves are of sausage mode.

**The problem formulation.** We use double-gradient MHD model (Erkaev et al., 2007) for finite thickness current sheet as mathematical model of flapping oscillations in this work. This model is based on the existence of a strong gradient \( \frac{\partial B_x}{\partial z} = J = \text{const} \) and a weak gradient \( \frac{\partial B_z}{\partial x} = \text{const} \) which provide rocking of the current sheet.

![Double-gradient model](image)

Figure 2: The double-gradient model (Erkaev, Semenov) for finite thickness current sheet, where \( 2\Delta \) is the thickness of the current sheet.

We observe a region with magnetic field very much elongated by X thus put \( \frac{\partial}{\partial x} = 0 \). There are two small parameters in our problem

\[
\nu = \frac{L_z}{L_x} \ll 1; \varepsilon = \frac{B_z}{B_x} \ll 1; \nu > \varepsilon,
\]

where \( L_x, L_z \) is the x and z-sizes of the current sheet, respectively, \( B_x, B_z \) is the magnetic field components.

Double-gradient model treats plasma as incompressible substance, since the effects coming from compressibility begin to appear at the velocities close to speed of sound (or Alfvén speed), while the velocities of flapping-waves are one order less. Therefore it is supposed that \( \rho = \text{const} \) and we can use the MHD system of equations for ideal incompressible plasma

\[
\begin{align*}
\rho (\frac{\partial \vec{v}}{\partial t} + (\vec{v}, \nabla)\vec{v}) &= -\nabla P + \frac{1}{4\pi} (\vec{B}, \nabla)\vec{B}; \\
\frac{\partial \vec{B}}{\partial t} + (\vec{v}, \nabla)\vec{B} &= (\vec{B}, \nabla)\vec{v} \\
(\nabla, \vec{v}) &= 0; (\nabla, \vec{B}) = 0.
\end{align*}
\]

We rely initial magnetic field as \( B_0 = (ax, 0, b + cx) \), where constant “c” presets a weak dependence of \( B_z \) on x and the constant “b” is the background \( B_z \) component. Below we will consider disturbances of magnetic field \( B \) with respect to the given initial field

\[
\vec{B}_{\text{full}} = \vec{B}_0 + \vec{B},
\]

\(| B | < B_0 \).
To decrease the number of equations let’s rewrite them with displacements vector $\xi$: $\ddot{v} = \frac{\partial^2 \xi}{\partial t^2}$. Then the components of the equation of plasmas motion will be written as:

\[
\begin{align*}
\rho \frac{\partial^2 \xi_x}{\partial t^2} &= \frac{1}{4\pi} (b^2 \frac{\partial^2 \xi_x}{\partial z^2} - ca \xi_x), \\
\rho \frac{\partial^2 \xi_y}{\partial t^2} &= -\frac{\partial p}{\partial y} + \frac{b^2}{4\pi} \frac{\partial^2 \xi_y}{\partial z^2}, \\
\rho \frac{\partial^2 \xi_z}{\partial t^2} &= -\frac{\partial p}{\partial z} + \frac{1}{4\pi} (b^2 \frac{\partial^2 \xi_z}{\partial z^2} - ca \xi_z);
\end{align*}
\]

It can be shown that the value $\Omega^2 = \frac{ca}{4\pi \rho}$ - double gradient frequency – is an important parameter of the matter. Finally, the system of equations is

\[
\begin{align*}
\frac{\partial^2 \xi_x}{\partial t^2} &= -\Omega^2 \xi_x, \\
\frac{\partial^2 \xi_y}{\partial t^2} &= -\frac{1}{\rho} \frac{\partial p}{\partial y}, \\
\frac{\partial^2 \xi_z}{\partial t^2} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} - \Omega^2 \xi_z; \\
\frac{\partial \xi_y}{\partial y} + \frac{\partial \xi_z}{\partial z} &= 0,
\end{align*}
\]

which may be solved with Fourier method. In Fourier space we get the equation of flapping for the displacement $\xi_z$:

\[
\frac{\partial^2 \xi_z}{\partial z^2} + k^2 \left( \frac{\Omega^2(z)}{\omega^2} - 1 \right) \xi_z = 0. \tag{1}
\]

For localized current sheet model under consideration

\[
\Omega(z) = \begin{cases} 
\Omega, & |z| < z_0; \\
0, & |z| > z_0,
\end{cases}
\]

where $z_0 = \frac{L_0}{2}$. Solution of the flapping equation (1) for $|z| < z_0$ is known:

\[
\xi_z = D_1 \cos \lambda z + D_2 \sin \lambda z,
\]

\[
\lambda^2 = k^2 \left( \frac{\Omega^2}{\omega^2} - 1 \right).
\]

Choice of the sinus or cosinus in this solution is specified by the mode of flapping oscillations. Even kink-mode corresponds to even function $\cos \lambda z$ and odd sausage-mode corresponds to sinus. Outside the current sheet the solution looks like

\[
\xi_z = C e^{\pm \lambda z}.
\]

The boundary condition implies that $\xi_z$ and $\frac{\partial \xi_z}{\partial z}$ are continuous across the structure, which leads to characteristic equations: $\tan \lambda z_0 = \frac{k}{\lambda}$ for kink mode, and $\tan \lambda z_0 = -\frac{\lambda}{k}$ for sausage mode.

Characteristic equations are solved numerically to get dispersion relations $\omega(k)$.

To return from Fourier space to the real space we need to make inverse Fourier trasformation with obtained dispersion.
Let’s set the initial disturbance as Gaussian $\xi(y) = e^{-y^2}$ ($z=0, \ t=0$), and contemplate only the kink-mode below, because the sausage mode may be obtained by similar processing. It is important that we set initial disturbance only for $z=0$. In the case if we set it for all $z$, all types of magnetospheres waves (including Alfvénic and fast sound oscillations) will appear and it will be difficult to separate surface flapping-waves. So we put $\xi(y)$ for $z=0$ and compute disturbances corresponding to flapping-oscillations for all other $z$.

$$\xi_z(t, y, z) = \frac{\text{Re}}{2\pi} \int_{-\infty}^{\infty} \xi_0 e^{i(\omega t - ky)} dk = \frac{1}{\sqrt{\pi}} \int_{0}^{\infty} e^{-x^2} \cos(\omega t) \cos(ky) \cos(z) dk.$$ 

Thus, we calculate how the shape of the current sheet changes with time. Next we can determine the other parameters of the model (velocity and magnetic fields components) from $\xi_z$ using MHD equations.

For normalization we use the following values: for distance – half width of the current sheet $z_0$, for magnetic field – value of background magnetic field $B_0$. Double gradient frequency $\Omega_0 = \sqrt{ca/4\pi\rho}$ is used as a unit of frequency, time measures in $T_0 = (\Omega_0)^{-1}$ and velocity in $v_0 = z_0 \Omega_0$.

**Results.** In spite of many simplifications – we used linear theory to describe nonlinear phenomenon – results obtained with this model well correlate with real data. In particular, basic frequency of the oscillations is equal to double gradient frequency ($\approx 1.3 \cdot 10^{-2}$ sec$^{-1}$), period $\approx 8$ minutes, amplitude $\approx 1R_e$, group velocity (with $k = 1$) is approximately equal to 57 km/s. Also oscillations damp fast (at distance about $1 - 2R_e$) along the Z-axis. (These values were obtained if we take $B_x = 30nT$ at sheets frontier and $B_z = 5nT$ at $x = 10R_e$.)

Let’s follow time evolution of the angle of the speed vector at fixed $z$ and $y$. Godograph represents the spiral: element of plasma makeup smoothly damping oscillations around the balance state. It can be seen that the direction of speed vectors rotation depend on the choice of the point of observation (in particular, it depends on the position relative to neutral sheet and to the source of the disturbance) and also on the type of observed oscillations.

**Figure 3:** Time evolution of the angle of the speed vector.
mode (kink or sausage). In addition double gradient frequency (central parameter of the model) can be
directly calculated from velocity of the rotation of this vector.
It is convenient to plot the variations of the angle of the speed vector in time as it is done in figure 3 (left
panel). The curves are inclined differently for the points located above or below the neutral sheet and for the
waves propagating to the dawn or to the dusk. Double gradient frequency is determined from the angle of the
curves inclination in this case.
The similar dependence was obtained for the Themis mission data on March 05, 2008, for the flapping waves
propagating to the dawn. Spacecraft were located under the sheet just as is represented in theoretical graphs
(left lower panel in the figure 3). There was series of inclined lines separated in experimental datas
(Sormakov, 2010) just like in the results of the modelling.
Inclined lines can be surely separated in data from P1 and P2 spacecraft (figure 1 and 3). In data from
spacecraft P4 lines were not distinguished supposedly because of much noisiness conditioned by nearness to
the Earth. Typical velocities of the flapping waves in this region are about 40 km/s, while all the
disturbances with the velocities under 15 km/s were recognized as noise, so the signal and the noise have the
similar values.
In theoretical model godoraph’s dependence on the choice of the point of observation and the mode of
oscillations, which was described above, is similar also for the vector of magnetic field, but this dependence
was not proved still experimentally.
Result obtained in this work gives reasons to propose that it is possible to use the double gradient model for
analysis of the experimental data. Time evolution of the angle of the vector of plasma velocity in the plane
YZ can be used for the choice between kink and sausage modes and also for the obtaining the main flapping
parameter – double gradient frequency. Main result of this work is conformity between the Themis mission
data and theoretical curves at the left lower panel in the figure 3, so we can surely determine mode of the
observed oscillations as kink.

It should be noted, however, that to determine the type of the mode we need to know the position of a
spacecraft relative to the neutral sheet and to the source of the disturbance, which can’t be done for all cases.
It is possible to define the position relative to the neutral sheet (by the sign of x-component of the magnetic
field), but it is not so easy for the location relating the source, which is the subject for further consideration.

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