IDENTIFICATION OF A USEFUL SIGNAL AND ITS LOW-FREQUENCY COMPONENT IN A NON-STATIONARY SEISMIC SIGNAL WITH HIGH NOISE LEVEL

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Abstract. The method for identification of the low frequency component in a non-stationary seismic signal with high noise level is based on advanced computer technology using the wavelet analysis of observed data. The main goal of this data processing system is finding the optimal fitting relation between the measured input and the required output. Application of signal wavelet decomposition allows one to identify those features in this signal which carry the information relevant to the object under study. As a result, we are enabled to examine the properties of a signal both in a physical (time) and a scale (frequency) spaces. The standard method for noise suppression is the elimination of noise components from the spectrum of the signal. In application to wavelet decomposition this can be realized in a straightforward manner by removing the detailing coefficients of high frequency levels. However, wavelets afford more ample possibilities for this kind of operation. The noise components, especially large random spikes, can also be treated as a set of local features. Specifying a threshold for their level and removing the detailing coefficients according to this threshold, one can not only diminish the noise level, but also assign the threshold limitations at several decomposition levels taking into account the concrete characteristics of noise and signals for different wavelet types. This makes it possible to design adaptive systems for noise elimination depending on noise features.

The method for identification of the low frequency component in a non-stationary seismic signal with high noise level is based on advanced computer technology using the wavelet analysis of observed data. The main goal of any data processing system is finding the optimal fitting relation between the measured input and the required output. The wavelet decomposition of a signal allows one to identify those features in this signal which carry the information relevant to the object under study.

The model of a noise-contaminated signal is usually assumed to be additive: $s(n) = f(n)+\eta(n)$ with a constant step over the argument n, where $f(n)$ is the useful information-rich component and $\eta(n)$ is the noise signal. The noise signal is usually assumed to be white noise at a prescribed level with zero mean. The procedure of identifying the low frequency component is done using orthogonal wavelets and involves the following steps:

- Wavelet decomposition of the signal $s(n)$ down to level N. The value N is determined by the frequency-dependent spectrum of the information part $f(n)$, which it is desired to leave in the series of fitting coefficients. The type and order of the wavelet can significantly affect the thoroughness with which the noise will be removed from the signal, depending both on the shape of $f(n)$ and on the correlation characteristics of the noise.
- Specifying the type and threshold levels of noise removal based on known prior information on noise or judging by certain criteria of the noise in the input signal. The threshold levels of noise removal may be flexible (depending on the number of decomposition level) or rigid (global).
- Modifying the coefficients which specify the detail available in the wavelet decomposition in accordance with the removal conditions specified.
- Reconstruction of $f(n)$ from the fitting coefficients and the modified detailing coefficients.

A continuous wavelet transform is described by the following relation:

$$CWT_X^{\psi}(t,s) = \psi(t,s) = \frac{1}{\sqrt{s}} \int x(t) * \psi\left(\frac{t-t'}{s}\right) dt',$$

$x(t)$ being is the signal and $\psi(t)$ is the window function.

The wavelet decomposition of a signal must, by analogy with the Fourier transformation, be such as to ensure complete informational equivalence of the new signal representation (the wavelet spectrum) and the temporal (dynamic, coordinate) representation, accordingly, to ensure uniqueness for signal decomposition and for signal reconstruction from wavelet spectra. This is however possible only by using orthogonal basis functions, which include among others also a rather limited number of orthogonal and biorthogonal wavelets. Unlike the Fourier transform, the wavelet transform ensures a two-dimensional imaging of the signal under study, the frequency and the time being treated as independent variables. As a result, we are enabled to examine the properties of a signal both in a physical (time) and a scale (frequency) spaces.

We now quote some wavelet properties of essential importance:

- Linearity. For vector functions this gives the result that the wavelet transform of a vector function is a vector consisting of the wavelet transforms of all components.
- Invariance under translation. Translation of a signal in time by *t0* gives a translation of the wavelet spectrum by *t0*.
- Scale invariance. Extension (compression) of a signal leads to extension (compression) of the associated wavelet spectrum.
- Differentiation. It comes to the same thing whether we differentiate a function or the analyzing wavelet.
- An analog of the Parceval theorem for orthogonal and biorthogonal wavelets. The energy of a signal can be computed via the coefficients of the wavelet transform.
- Localization. A wavelet must be continuous, integrable, have a compact support, and be localized both in time (space) and in frequency. If the wavelet is narrowed in space, its "average" (dominant) frequency becomes higher, the wavelet spectrum is shifted toward higher frequencies and becomes broader. The process must be linear, the narrowing doubling the dominant frequency and the spectrum width. A wavelet function may be considered sufficiently well localized, if local features of signals can be detected within the wavelet support at the level of regional variations and trend, a zero increase in the signal's constant component with zero value of the wavelet frequency-dependent spectrum at $\omega=0$, and localization of the wavelet spectrum as a banded spectrum centered at a certain (dominant) frequency ω_0 of the wavelet function. In order to be able to ignore regular polynomial components of a signal and to analyze small-scale fluctuations and features of higher orders, we generally require wavelets of the *m*-th order.
- Finiteness. The necessary and sufficient condition: $\|\Psi(t)\|^2 = \int_{-\infty}^{\infty} |\Psi(t)|^2 dt < \infty$. Estimation of fairly good finiteness and localization can be carried out using the expressions $|\Psi(t)| < 1/(1+|t|n)$ or $|\Psi(\omega)| < 1/(1+|\omega_0|n)$
- Self-similarity. The shape of all basis wavelets $\Psi_{ab}(t)$ must be similar to that of the parent wavelet Ψ(t), that is, must remain the same under translation and scaling (extension/compression), and must have the same number of oscillations.

The properties of a continuous wavelet transform can be extended to cover the multidimensional and discrete cases.

The result of applying the wavelet transform to a one-dimensional series of values (signal) is a two-dimensional array of the coefficients *C*(*a,b*). The distribution of these values in the (*a,b*) space - temporal scale, temporal localization - provides information on temporal changes of the relative contribution in the signal of wavelet components of different scales and is called the spectrum of the wavelet transform coefficients, the scale-time (frequency-time) spectrum or simply the wavelet spectrum. The spectrum *C*(*a,b*) of a one-dimensional signal is a surface in a three-dimensional space.

The choice of the analyzing wavelet is largely governed by the information one hopes to extract from the signal. Taking into consideration typical features of the various wavelets in the time and frequency spaces, one can examine signals to be analyzed to identify in these various properties and features that cannot be perceived in plots of the signals, especially in the presence of strong noise. The choice of the analyzing wavelet and the decomposition depth depends on the properties of the signal to be analyzed. More smooth wavelets produce a more smooth fit to the signal, while "short" wavelets afford a clearer view of peaks in the fitted function. Decomposition depth affects the scale of details eliminated. Greater decomposition depth allows the model to subtract noise of ever higher levels, but the associated smoothing affects not only the noise, but also some local features of the signal. For the wavelet transform one has to compute a series of fitting and detailing wavelet coefficients.

The noise component of a signal is mostly reflected in the detailing coefficients D_j , consequently, it is these coefficients which are being affected by the processing. Besides, the noise component is a signal with a smaller absolute value than the signal of interest. For this reason noise is eliminated by making the coefficient values below some threshold actually zero. The choice of the background threshold value affects the quality of noise elimination as estimated by the signal/noise ratio. Choosing a small threshold will preserve the background in the detailing coefficients, and so contributes little to a higher signal/noise ratio. With large threshold values one can miss those coefficients which carry significant information. The search for the optimal threshold value reduces to finding the greatest signal/noise ratio for the smallest translation of the reconstructed signal.

The standard method for noise suppression is the elimination of noise components from the spectrum of the signal. In application to wavelet decomposition this can be realized in a straightforward manner by removing the detailing coefficients of high frequency levels. However, wavelets afford more ample possibilities for this kind of operation. The noise components, especially large random spikes, can also be treated as a set of local features. Specifying a threshold for their level and removing the detailing coefficients according to this threshold, one can not only diminish the noise level, but also assign the threshold limitations at several decomposition levels taking into account the concrete characteristics of noise and signals for different wavelet types. This makes it possible to design adaptive systems for noise elimination depending on noise features.

The task is dealt with in four steps:

1. The raw signal is decomposed on the wavelet basis.
2. A threshold noise value is selected for each decomposition level;
3. Threshold filtering of the detailing coefficients is carried out;
4. The signal is reconstructed.

This task was solved in the MATLAB programming shell. We used the entropy-log-energy criterion to choose the optimal wavelet decomposition.

The entropy of the original signal is at the maximum because of noise contamination. As the level of wavelet decomposition increases, the entropy diminishes down to the minimum value corresponding to the optimal level of wavelet decomposition of the original signal. Noise was eliminated by the Birge-Massart method: a threshold value was determined for the K criterion to be applied to the detailing coefficients. The coefficient values below the threshold were set equal to zero and the value of K was subtracted from the values of the remaining coefficients. The optimal value of the K test was selected by the minimum entropy-log-energy principle. The choice of the optimal wavelet which produced the best level of wavelet elimination of noise was by the criterion of the ratio between the entropy of the original signal and that with noise eliminated.

At the first step we selected the best tree for each wavelet type by the entropy-log-energy criterion. We used orthogonal compactly supported wavelets: Daubechies (db), symplets (sym), coiflets (coif). Entropy was calculated for each level of signal wavelet decomposition at different values of the threshold criterion $\boldsymbol{K}$. Figure 1 shows plots of the original signal, the tree of the package wavelet, and the wavelet representation of the signal. The processing was carried out by the Daubechies wavelet (db4) with decomposition level 4. Figure 2 shows the initial data for elimination of the noise component. Figure 3 shows the original process and that with the noise eliminated. It can be seen from these plots that the procedure under discussion does not cope with the elimination of the detailing coefficients at the lowest level, these being put equal to zero externally. The result is the noise-eliminated process shown in Fig. 4.

We used the fast wavelet transform to eliminate noise. It should always be remembered that, when the total signal size is M values and the maximum decomposition level is N, then $M/2N$ must be integer in order to ensure the normal functioning of the fast wavelet transform; this ensures an integer number of coefficients at the last decomposition level. If that requirement is not met, it is recommended to supplement the array of data values by zeroes or any other values.

CONCLUSIONS

In this article is shown that application as systems of a preliminary adaptive filtration of input seismic signals of systems using fast wavelet transformation allows increasing essentially a ratio signal/noise that in turn essentially facilitates the subsequent classification of the allocated signals by an artificial neural network. Thus it is always necessary to consider that if the full size of a signal makes M-counting, and the maximum level of decomposition is equal to N, for ensuring normal work of fast wavelet-transformation the relation of M/2N should be an integer that provides an integer of factors at the last level of decomposition. If this condition isn't carried out, recommended to supplement the dimension of counts with zero or any other values. At preliminary tests the system proved to be as the extremely effective and not demanding big computing capacities that will allow applying further it in the preliminary earthquake alarm systems working in real time.

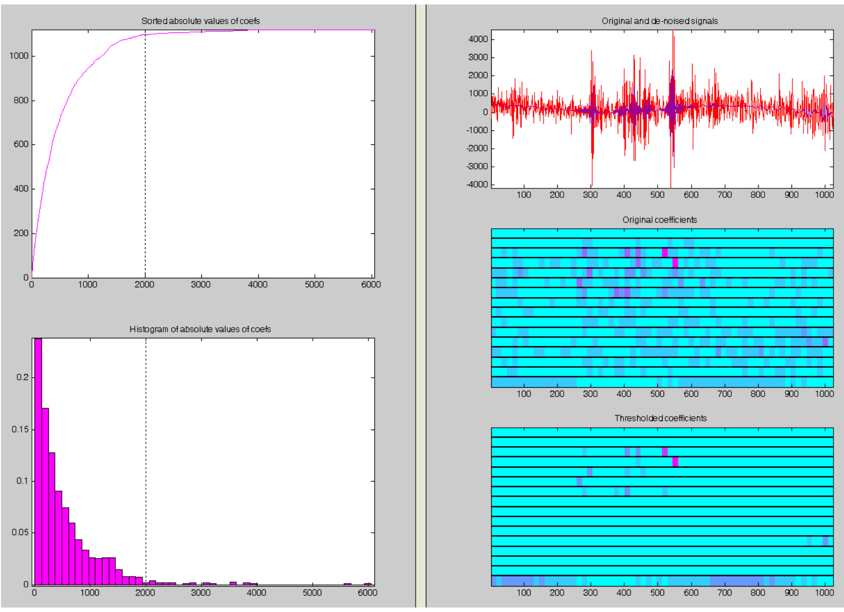

Fig. 1. Wavelet processing of an input signal.

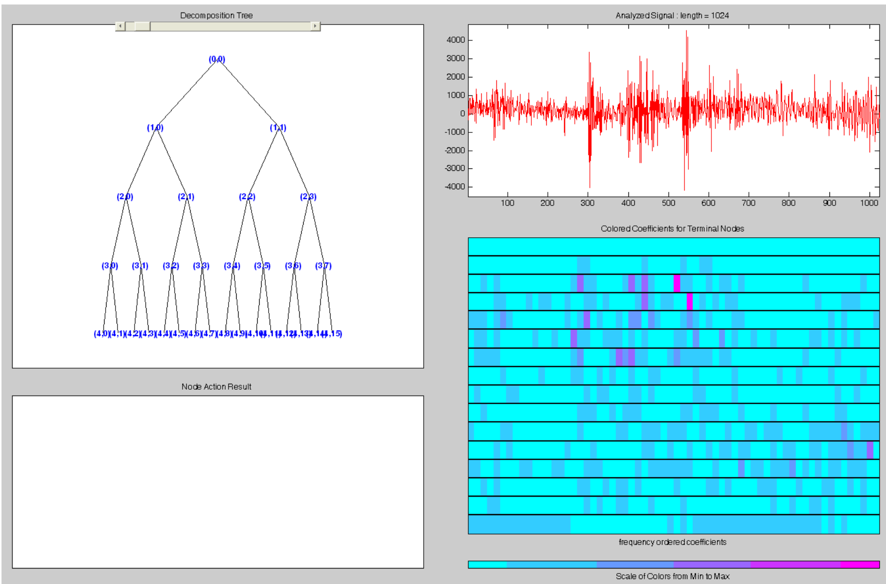

Fig. 2. Eliminating the noise component.

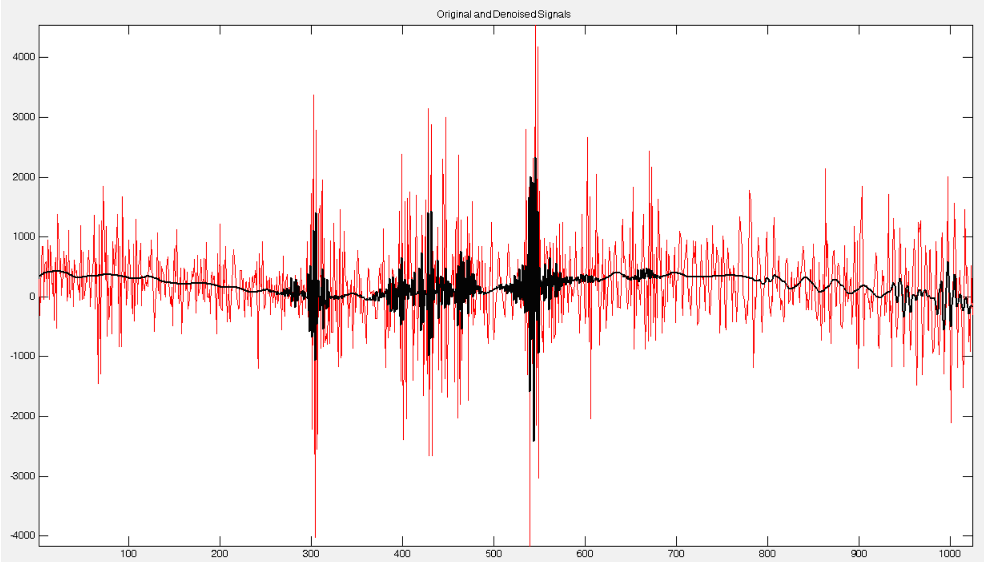

Fig. 3. The original process and that with noise eliminated.

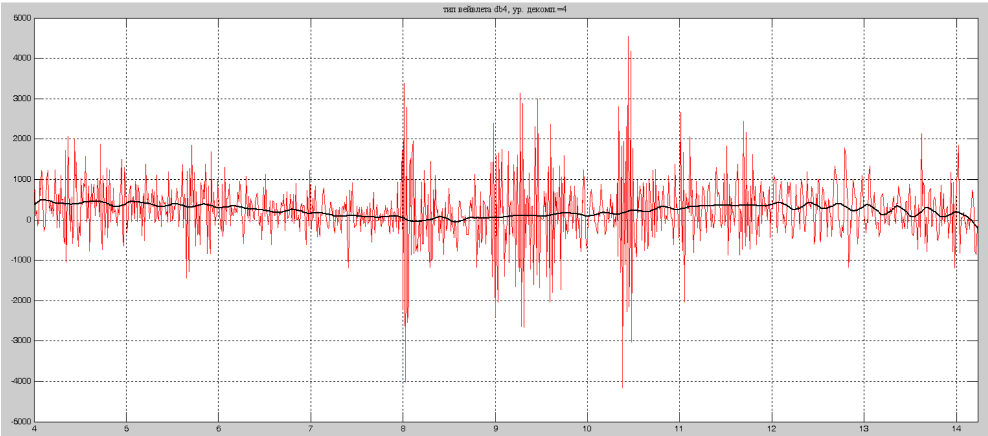

Fig. 4. The original process and the final process with noise eliminated.

ACKNOWLEDGEMENTS

This work is conducted at the Institute of Earthquake Prediction Theory and Mathematical Geophysics, Russian Academy of Sciences (former IIEPT RAS) together with specialists at the Federal Unitary Enterprise “State Research Institute of Aviation Systems” with the financial support of the European Community by the International Science and Technology Center (ISTC). We would like to thank Dr. Shigeki Horiushi, Senior Researcher at the National Research Institute for Earth Sciences and Disaster Prevention and Prof. Th. Camelbeek, Head of Seismology Department, Royal Observatory of Belgium for helpful discussions and the data they lent us. We also acknowledge useful consultations with Dr. P.G. Krug on neural network organization.

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