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NUMERICAL 2-D MHD MODELING OF THE SOLAR WIND
BETWEEN THE SUN AND THE EARTH

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ABSTRACT

An attempt to infer parameters of the solar corona and of the solar wind by means of a numerical, self-consistent, MHD simulation with boundary conditions for magnetic field given from the observations of the large-scale magnetic field at the Sun is made. A two-region, planar (the ecliptic plane is assumed) model for the solar wind flow is considered. The region I of transonic flow is assumed to cover the distances from the solar surface up to 10 \( R_S \) (\( R_S \) is the radius of the Sun), and the region II of supersonic, superalfvénic flow extends between the 10 \( R_S \) and the Earth's orbit. The treatment of the problem for the region I, as a mixed initial-boundary value MHD one, and the solution procedure are similar to those discussed by Endler (1971) and Steinolfson et al., (1982): a steady-state solution is searched for as a relaxation to the dynamic equilibrium of an initial state with time-independent boundary conditions. To obtain a solution to initial value problem in region II with the initial distribution of dependent variables at 10 \( R_S \) deduced from the solution for region I, the numerical scheme similar to that used by Pizzo (1978, 1982) was applied. The solar rotation is taken into account for the region II and, hence, the interaction between fast and slow solar wind streams is self-consistently treated. As a test example for the proposed formulation and numerical technique, a solution for the problem similar to that discussed by Steinolfson et al. (1982) is obtained. To demonstrate the applicability of our scheme to experimental data, solar magnetic field observations at Stanford University for Carrington rotation 1682 were used to prescribe boundary conditions for the magnetic field at the solar surface. The steady-state solution appropriating the given boundary conditions was obtained for region I and then traced to the Earth's orbit through the region II. Comparing the calculated and observed by satellites the solar wind velocity, the radial magnetic field, and the number
density, one can see that general trends during the solar rotation are fairly good reproduced.

1. MODEL FORMULATION

We assume that the solar wind plasma can be described by single-fluid, non-dissipation, magnetohydrodynamic equations in equatorial plane of spherical coordinate system. In region I we take into account pressure gradient, gravitational and magnetic forces for the momentum equation and neglecting the solar rotation. The independent variables in the governing system of time-dependent, MHD system of equations for the region I are the time t, the radius r and the longitude φ. The six dependent variables are the density ρ, radial $B_r$, and azimuthal $B_\phi$ components of the magnetic field, radial $u_r$ and azimuthal $u_\phi$ components of velocity, and pressure P. As to the region II, we assume supersonic, supralvénic, steady-state flow and take into account the solar rotation. Hence, there are two independent variables in the region: the radius r and the longitude φ. The following condition was used to reduce the number of the differential equations in the region II. We assume that there is no steady electric field in the rotating frame (Pizzo, 1982), thus, the vectors of velocity and magnetic field are parallel: $B_\phi/B_r = v_\phi/v_r$, and we can calculate, for instance, azimuthal component of magnetic field $B_\phi$ from this relation.

As an initial state for hydrodynamic variables, it is assumed a spherically symmetric polytropic flow given by a Parker-type solution of 1-D HD equations. The following reference values discussed by Stenoljofson et al. (1982) were used at 1 $R_S$: proton density $N_o = 2.25 \times 10^8$ cm$^-3$ and temperature $T_o = 1.8 \times 10^6$ K. The polytropic index γ is assumed to be equal to 1.05. The radial velocity $u_r$ at 1 $R_S$ (10 $R_S$) is of about 8.1 (266) km s$^{-1}$. For region II we assume for γ the value of 1.17 that has been deduced from the solar wind observations by Sittler and Scudder (1980).

To demonstrate the technique outlined in the present study, we have chosen the solar magnetic data for Carrington rotation 1682 (Hoekema and Sherrer, 1985). The rotation covers the time interval near the maximum phase of the 21 solar cycle. The epoch of high solar activity, when the heliospheric current sheet (HCS) is strongly wrapped, has been chosen for analysis because of
two-dimensional formulation of the problem. The formulation implies
the neglecting by meridional gradients and, hence, the greater the
HCS inclines to the equator, the better the 2-D approximation is
worked out.

To compute initial distribution for $B_r$ and $B_\phi$, we used the
magnitudes of spherical harmonics (up to third order) calculated by
Hoeksema and Sherrrer (1985) using a potential magnetic field model
for spherical shell between 1 $R_S$ and the 'source surface' at 2.5
$R_S$. On and outside the shell we assume that the field is pure
radial and decreased as $r^{-2}$. The latter follows the divergence-free
condition.

The method employed to trace the temporal evolution of the
initial state by advancing the solution from $t$ to $t+\Delta t$ for region I
and to step the solution from $r$ to $r+\Delta r$ in region II is highly
efficient numerical scheme developed by MacCormac (1969).

The two-dimensional grid, applied in region I consists of 38
points in longitude with a spacing of 10° (36 routine points and
two auxiliary ones to conveniently assign periodic boundary
conditions). To reduce the number of grid points along the radius
and at the same time to account for the most sharp gradients near
the solar surface, the radial coordinate $r$ was transformed to a new
logarithmic one $r_1 = \ln (r/r_0)$, where the parameter $r_0$ is assumed to
be equal to 1 $R_S$. If the grid points are equally spaced along $r_1$,
grid spacing along $r$ increases proportionally to $r$ ($\Delta r = r_1 \Delta r_1$).

To maintain the numerical stability of computations in region I
we applied the commonly used (Steinolfson et al., 1982, Han,
1977) artificial diffusion scheme developed by Lapidus (1967). We
chose the time (or radial) step size from the Courant condition,
taking into account the limitations to the step size imposed by
explicit diffusion.

We use the following combination of boundary conditions for
the inner boundary. The radial magnetic field $B_r$ is assumed to be
fixed at the initial values derived from the observations. The
radial velocity $u_r$ is also fixed at the initial value if the flow
at the adjacent (along $r$) point is antisunward, or can drop to zero
if the flow at the adjacent point is turned to the Sun. To
prescribe the boundary values for the pressure $P$, we use the
following procedure. Although the radial velocity at the boundary
is fixed ($u_r^0$), a value $u_r^E$ for radial velocity is calculated from
the values at two grid points adjacent to the boundary using a linear extrapolation procedure. Maintaining the fixed value $u^0_r$ at the surface, we assume that the radial moment, which is approximated using difference $\Delta u_r = u^e_r - u^0_r$ as $\rho \Delta u^2_r/2$, is artificially converted to thermal energy (cf. Han, 1977) and, hence, the modified pressure $P'$ is given by

$$P' = P + (\gamma - 1)\rho \frac{\Delta u^2_r}{2} \left(1 - \frac{\Delta u_r}{|\Delta u_r|}\right).$$

The boundary values for azimuthal field $B_\phi$ are linear extrapolated from the values at two grid points adjoined to the boundary. The azimuthal speed $u_\phi$ is calculated from the condition that the vectors of the magnetic field and velocity are parallel at the inner boundary. The density $\rho$ are determined from the condition $P'/\rho^{\gamma} = P_0'/\rho_0^{\gamma}$, where $P_0$ and $\rho_0$ are the initial values of pressure and density, respectively, at the solar surface. Emphasize that in the present study we do not assume any longitudinal variation of boundary values of hydrodynamic parameters. Hence, the longitudinal effects which will be described below are caused exclusively by the influence of non-homogeneous magnetic field.

Both regions I and II have artificial computational boundaries at $\phi = 0$ and $\phi = 360^\circ$. At the boundaries periodic boundary conditions are prescribed. The region II has no more boundaries, but region I has an another computational boundary at the top radial limit (10 $R_S$). At the boundary, where the flow is assumed to be supersonic and superalfvénic, a linear extrapolation all of the conservation variables are used to assign extreme values to dependent variables.

2. NUMERICAL RESULTS AND DISCUSSION

The initial state and the steady state, resulted from numerical simulation after 32 hours of relaxation from the initial state in region I, is shown in Fig. 1 as a planar map of magnetic field lines and flow velocities. The flows are approximately aligned along the magnetic field lines, as it has to be for steady state in approximation of dissipationless flow. However, the artificial diffusion, applied implicitly by difference method itself and explicitly by Lapidus scheme as well, allows the divergence of the magnetic field lines from the flow ones that is why one can see in Figure 1 such deviations. The azimuthal
variations of the radial velocity and thermodynamic parameters at 10 $R_S$ are shown in Figure 2. It demonstrates three regions of higher pressure which are extended above the closed field regions in the flow field. One can see that pressure and density curves are similar to each other (this is a sequence of the small value of polytropic index $\gamma = 1.05$) and both are inverted with respect to the velocity curve.

We do not present the intermediate stages of the evolution since they have little physical meaning (Suess, 1983). However, the time, which system take to approach the steady state, does have meaning as characteristic time of relaxation of the solar corona with respect to large-amplitude perturbations (Suess, 1983). Our computations support a value of about 16–24 hours for the relaxation time in the coronal region up to 5–10 $R_S$ inferred from their studies by Endler (1971) and by Stenolfson et al. (1982).

The values of dependent variables at the top boundary of region I (10 $R_S$) were used to proceed the solution up to the Earth's orbit on the longitude grid with a spacing of 10° (this takes only about 20 steps along radial coordinate). Since the solar rotation is taken into account, the interaction of the solar wind streams of different velocities is self-consistently reproduced.

The Figure 3 shows the results of the present simulation for Carrington rotation 1682 at the orbit of the Earth. It presents by dashed lines the calculated solar wind velocity $V$, the radial magnetic field $B_X$ ($B_X = -B_R$ if solar ecliptic (GSE) coordinates are assumed), the number density $N$, and the solar wind temperature $T$. The same parameters observed by satellites (King, 1986) are shown by the full line. One can see that reasonably good agreement of general trends in the observed and computed velocity, radial magnetic field, density and temperature. However, the computed variation in the solar wind velocity is of about 30 km s$^{-1}$ instead of 300 km s$^{-1}$ observed. It seems to be a sequence of spherically symmetric, radial independent heat addition which is implied by polytropic approximation used with non-adiabatic value of $\gamma$. The other result of the polytropic assumption is that the calculated numerical density is still in about of order of magnitude higher comparing with the observed one that is the typical difficulty of Parker-type solutions for the solar wind.

The proposed numerical technique is thought to be of some
value for interpretation of solar-terrestrial relations and in predictions of the geomagnetic activity which is controlled by the solar wind and the IMF parameters (Pudovkin et al., 1980, Usmanov, 1990). But these problems extend beyond the scope of the present paper and will be addressed in future studies.

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FIGURE CAPTURES

Fig. 1: Initial (a) and resulted steady (b) states for simulation of CR 1682 in region I: the magnetic field configuration superimposed on flow in format similar to that used by Stenolfson et al. (1982). The arrows point in the direction of the velocity and their length is proportional to the velocity magnitude at their base. The sonic and Alfvén curves are shown by the dashed and dashed-dotted line, respectively.

Fig. 2: Results of simulation for CR 1682 in region I: the azimuthal variation at 10 Rs of the radial velocity, pressure, density, and temperature T in the steady state.

Fig. 3: Results of simulation for CR 1682 at the Earth's orbit: the curves plotted as dashed lines are the calculated solar wind velocity V, the radial magnetic field B_r, the number density N, and the solar wind temperature T. The solar wind parameters observed by spacecraft are shown by the solid lines. Note the different scales for the observed and computed variables (excepting for the magnetic field).
Figure 1
Figure 2

Figure 3