INTERPLANETARY MAGNETIC FIELD STRUCTURE AND
SOLAR WIND PARAMETERS AS INFERRED FROM SOLAR
MAGNETIC FIELD OBSERVATIONS AND BY USING A
NUMERICAL 2-D MHD MODEL

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Abstract. An attempt is made to infer parameters of the solar corona and the solar wind by means of a numerical, self-consistent MHD simulation. Boundary conditions for the magnetic field are given from the observations of the large-scale magnetic field at the Sun. A two-region, planar (the ecliptic plane is assumed) model for the solar wind flow is considered. Region I of transonic flow is assumed to cover the distances from the solar surface up to $10 \ R_S$ ($R_S$ is the radius of the Sun). Region II of supersonic, super-Alfvénic flow extends between $10 \ R_S$ and the Earth’s orbit. Treatment for region I is that for a mixed initial-boundary value problem. The solution procedure is similar to that discussed by Endler (1971) and Steinolfson, Suess, and Wu (1982): a steady-state solution is sought as a relaxation to the dynamic equilibrium of an initial state. To obtain a solution to the initial value problem in region II with the initial distribution of dependent variables at $10 \ R_S$ (deduced from the solution for region I), a numerical scheme similar to that used by Pizzo (1978, 1982) is applied. Solar rotation is taken into account for region II; hence, the interaction between fast and slow solar wind streams is self-consistently treated. As a test example for the proposed formulation and numerical technique, a solution for the problem similar to that discussed by Steinolfson, Suess, and Wu (1982) is obtained. To demonstrate the applicability of our scheme to experimental data, solar magnetic field observations at Stanford University for Carrington rotation 1682 are used to prescribe boundary conditions for the magnetic field at the solar surface. The steady-state solution appropriate for the given boundary conditions was obtained for region I and then traced to the Earth’s orbit through region II. We compare the calculated and spacecraft-observed solar wind velocity, radial magnetic field, and number density and find that general trends during the solar rotation are reproduced fairly well although the magnitudes of the density in comparison are vastly different.

1. Introduction

The solar magnetic field is believed to play a crucial part in the formation of the solar corona and the solar wind structure. It follows that magnetic pressure near the Sun essentially dominates the dynamic and thermal pressure when the Alfvén Mach number $< 1.0$. Obviously, the coronal and interplanetary magnetic fields are the photospheric magnetic fields stretched by the expanding plasma of the solar corona. Availability of large-scale magnetic field data at the solar photosphere (Hoeksema and Scherrer, 1985; Grigoryev et al., 1986), combined with the attractive possibility of MHD modeling of interplanetary medium structure (see, e.g., the review by Dryer, Smith, and Wu, 1988), suggests an idea to try to infer the interplanetary magnetic field (IMF) and the solar wind parameters between the Sun and the Earth. As suggested in that review, the technique is a numerical, self-consistent simulation of interplanetary medium dynamics using solar magnetic field observations as boundary conditions. Note here that it is also possible

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to account for the interplanetary medium structure using a prescribed field geometry, as has been done by Durney and Pneuman (1975), or using a kinematic approach (Hakamada and Akasofu, 1982), but a self-consistent treatment is preferable due to alternative domination of the competitive factors of magnetic and dynamic pressures in the solar wind dynamics.

The first approach, which has been intensively used during the past two decades to simulate interplanetary plasma and magnetic field dynamics, has dealt with hydrodynamic (HD) or magnetohydrodynamic (MHD), steady-state or time-dependent, one-, two-, or three-dimensional flows which are supersonic and super-Alfvénic. In such cases arbitrary initial conditions may be prescribed on a surface which is located outside the outermost critical point. Typically, some initial conditions are prescribed on a sphere (or a circle for the 2-D case) of radius 0.08–0.16 AU (e.g., Nakagawa and Wellick, 1973; Wu, Han, and Dryer, 1979; Wu, Dryer, and Han, 1983; Pizzo, 1978, 1980, 1982; Han, Wu, and Dryer, 1988) and, by means of an explicit numerical scheme, one can solve the initial-value Cauchy problem by integrating the governing equations along the hyperbolic radial coordinate for the steady-state or along the hyperbolic time-coordinate in a time-dependent case.

The question is: what boundary conditions should be prescribed at the initial level? The most natural answer is to set up the conditions from a solution of the transonic problem of solar wind structure formation near the Sun which is the second approach used to simulate the interplanetary medium dynamics. Several papers devoted to this problem were initiated in pioneering work by Pneuman and Kopp (1971) who obtained, for the first time, a self-consistent, steady-state solution for the 2-D magnetic field and mass flow configuration such as that in a coronal streamer. Pneuman and Kopp (1971) restricted their solution to the isothermal case and used an iterative procedure to obtain a solution describing the resulting steady state. In order to do this solution, they had to make an assumption regarding the nature of the cusp at the top of the closed region. Robertson (1983) and Cuperman, Ofman, and Dryer (1990) relaxed the assumption of a constant temperature and considered thermally conductive flows. Pisanko (1985a, b) constructed a three-dimensional solution to the Pneuman–Kopp problem. Robertson (1983) and Pisanko (1985a, b) both used the Pneuman–Kopp iterative technique. Cuperman, Ofman, and Dryer (1990) improved this technique by indicating the correct procedure for closure and, thus, self-consistency.

Another treatment of the Pneuman–Kopp problem was proposed by Endler (1971). Endler (1971) suggested that the problem be treated as a mixed initial-boundary value one for a time-dependent MHD system of equations. He suggested a time-relaxation technique whereby one searches for a steady-state solution as a result of temporal evolution of an initially prescribed plasma-magnetic field configuration with given boundary conditions. This computational treatment allows the removal of the assumption of the cusp location and uses the same explicit methods as those applied for supersonic, super-Alfvénic flows due to the hyperbolic nature of the governing system of equations with respect to time. The same relaxation procedure was used by Steinolfson, Suess, and Wu (1982; hereafter referred to as SSW), Wu and Wang (1991),
and Linker, Van Hoven, and Schnack (1990). SSW obtained a set of solutions for the non-isothermal case (Endler (1977) assumed a constant temperature distribution) and for a number of plasma-β values, i.e., for a range of magnetic field strengths at the Sun. Linker, Van Hoven, and Schnack (1990) and Wu and Wang (1991) constructed a 3-D solution for a coronal streamer.

In the present work we tried to construct the full solution to a 2-D MHD steady-state problem applicable to the solar wind and the IMF structure between the solar surface and the Earth’s orbit. This is, to our knowledge, the first attempt to bridge a gap between the above two specified approaches to MHD simulation of interplanetary medium structure in the two-dimensional steady-state formulation.

The next question is: how should one prescribe boundary conditions near the Sun’s surface? It is natural to do this by using measurements of the large-scale photospheric magnetic field made at Stanford University (Hoeksema, 1984; Hoeksema and Scherrer, 1985) or at Sayan Observatory (Grigoryev et al., 1986). There is the following circumstance that simplifies the boundary specifications for other variables. It is necessary to specify some boundary values to all dependent variables (not violating, however, the solenoidality conditions discussed by Yeh and Dryer, 1985) if an initial surface is located in a region of supersonic and super-Alfvénic flow regime. Specification of the boundary conditions near the Sun has an advantage, however, since all of the dependent variables need not be prescribed arbitrarily. Some of them can be determined from flow characteristics near the boundary from the so-called compatibility relations (e.g., Endler, 1971; Han, 1977; Wang, Hu, and Wu, 1982). Alternatively, a procedure of extrapolation to the boundary as a substitute for the compatibility relations may be used (Steinolfson and Nakagawa, 1976).

2. Model Formulation

2.1. Basic equations

We use the following physical model to infer parameters of the solar corona and of the solar wind by means of 2-D numerical MHD modeling. We suppose that the Earth is located in the ecliptic plane and neglect the deviation of about 7.5° of the ecliptic plane from the solar equatorial plane. The domain of our interest, which extends from the solar surface up to the Earth’s orbit, is divided into two regions. The first one, denoted below as region I, extends from the solar surface up to 10 Rs and the second region, II, from 10 Rs up to the Earth’s orbit (215 Rs). Both regions cover the full range of longitudes from 0 to 360°. We assume that the solar wind plasma can be described by dissipationless, single-fluid magnetohydrodynamic equations in the equatorial plane of the spherical-coordinate system. Taking into account pressure gradient, gravitational and magnetic forces for the momentum equation, and neglecting the solar rotation, we can write the governing time-dependent MHD system of equations for region I in the following
non-dimensional form:

\[
\frac{1}{S_h} \frac{\partial}{\partial t} (r^2 \rho) = - \frac{\partial}{\partial r} (r^2 \rho u_r) - r \frac{\partial}{\partial \phi} (\rho u_\phi), \quad (1a)
\]

\[
\frac{1}{S_h} \frac{\partial}{\partial t} (r B_r) = - \frac{\partial}{\partial \phi} (u_\phi B_r - u_r B_\phi), \quad (1b)
\]

\[
\frac{1}{S_h} \frac{\partial}{\partial t} (r B_\phi) = \frac{\partial}{\partial r} \left[ r (u_\phi B_r - u_r B_\phi) \right], \quad (1c)
\]

\[
\frac{1}{S_h} \frac{\partial}{\partial t} \left\{ r^2 \rho \left[ \frac{P E_u}{(\gamma - 1) \rho} + \frac{1}{2} (u_r^2 + u_\phi^2) + V_{ar}^2 + V_{a\phi}^2 \right] \right\} = - \frac{\rho u_r}{F_r} - \frac{\partial}{\partial r} \left\{ r^2 \rho \left[ u_r \left( \frac{\gamma P E_u}{(\gamma - 1) \rho} + \frac{1}{2} (u_r^2 + u_\phi^2) \right) + V_{a\phi} (u_r V_{a\phi} - u_\phi V_{ar}) \right] \right\} - \frac{\partial}{\partial \phi} \left\{ r \rho \left[ u_\phi \left( \frac{\gamma P E_u}{(\gamma - 1) \rho} + \frac{1}{2} (u_r^2 + u_\phi^2) \right) - V_{ar} (u_r V_{a\phi} - u_\phi V_{ar}) \right] \right\}, \quad (1d)
\]

\[
\frac{1}{S_h} \frac{\partial u_r}{\partial t} = - u_r \frac{\partial u_r}{\partial r} - u_\phi \frac{\partial u_r}{\partial \phi} + \frac{u_r^2}{\rho} - \frac{E_u}{\rho} \frac{\partial \rho}{\partial r} - \frac{1}{F_r r^2} + \frac{1}{M_a^2 \rho} \left( \frac{B_\phi}{r} \frac{\partial B_r}{\partial \phi} - B_r \frac{\partial B_\phi}{\partial r} - \frac{B_\phi^2}{r} \right), \quad (1e)
\]

\[
\frac{1}{S_h} \frac{\partial u_\phi}{\partial t} = - u_r \frac{\partial u_\phi}{\partial r} - u_\phi \frac{\partial u_\phi}{\partial \phi} - u_\phi u_r - \frac{E_u}{\rho} \frac{\partial \rho}{\partial r} + \frac{1}{M_a^2 \rho} \left( B_r \frac{\partial B_\phi}{\partial r} - \frac{B_r}{r} \frac{\partial B_\phi}{\partial \phi} + \frac{B_\phi B_r}{r} \right), \quad (1f)
\]

where the independent variables are the time \( t \), the radius \( r \), and the longitude \( \phi \). The six dependent variables are the density \( \rho \), radial \( B_r \) and azimuthal \( B_\phi \) components of magnetic field, radial \( u_r \) and azimuthal \( u_\phi \) components of velocity, and pressure \( P \). \( V_{ar} = B_r / \sqrt{M_a^2 \rho} \) and \( V_{a\phi} = B_\phi / \sqrt{M_a^2 \rho} \) are radial and azimuthal Alfvén velocity components. The constants of normalization are the Strouhal number \( S_h = \tilde{u} / \tilde{L} \), the Froede number \( F_r = (\tilde{u}^2 \tilde{L}) / (GM_s) \), the Mach Alfvén number \( M_a = \tilde{u} \sqrt{4 \pi \tilde{\rho} / \tilde{B}} \), and the Euler number \( E_u = \tilde{P} / (\tilde{\rho} \tilde{u}^2) \), where \( \tilde{u} \), \( \tilde{T} \), \( \tilde{L} \), \( \tilde{P} \), \( \tilde{\rho} \), and \( \tilde{B} \) are units of velocity, time, length, pressure, density, and magnetic field, respectively. \( G \) is the universal gravitational constant; \( M_s \) is the solar mass.

As to region II, we assume supersonic, super-Alfvénic, steady-state flow and take into account the solar rotation. Hence, there are only two independent variables in the
region: the radius $r$ and the longitude $\varphi$. The governing equations for region II are:

$$\frac{\partial}{\partial r} (r^2 \rho u_r) = -r \frac{\partial}{\partial \varphi} (\rho v_\varphi), \quad (2a)$$

$$\frac{\partial}{\partial r} \left[ r^2 \rho \left( u_r^2 + \frac{P}{\rho} E_u - \frac{1}{2} (V_{ar}^2 - V_{a\varphi}^2) \right) \right] =$$

$$= -r \frac{\partial}{\partial \varphi} \rho (u_r v_\varphi - V_{ar} V_{a\varphi}) + r \rho \left( u_\varphi^2 - \frac{1}{F_r} + \frac{2P}{\rho} E_u + V_{ar}^2 \right), \quad (2b)$$

$$\frac{\partial}{\partial r} \left[ r^2 \rho (u_r u_\varphi - V_{ar} V_{a\varphi}) \right] = -r^2 \frac{\partial}{\partial \varphi} \rho \left[ u_\varphi v_\varphi + \frac{P}{\rho} E_u + \frac{1}{2} (V_{ar}^2 - V_{a\varphi}^2) \right], \quad (2c)$$

$$\frac{\partial}{\partial r} (r^2 B_r) = -r \frac{\partial B_\varphi}{\partial \varphi}, \quad (2d)$$

$$\frac{\partial}{\partial r} \left\{ r^2 \rho \left[ u_r \left( \frac{\gamma P E_u}{(\gamma - 1) \rho} + \frac{1}{2} (u_r^2 + u_\varphi^2) \right) - V_{ar} V_{a\varphi} \frac{r}{R_0} \right] \right\} =$$

$$= -r \frac{\partial}{\partial \varphi} \rho \left[ v_\varphi \rho \left( \frac{\gamma P E_u}{(\gamma - 1) \rho} + \frac{1}{2} (u_r^2 + u_\varphi^2) \right) +$$

$$+ \frac{r}{R_0} \rho \left( \frac{P}{\rho} E_u + \frac{1}{2} (V_{ar}^2 - V_{a\varphi}^2) \right) \right] - \frac{\rho u_r}{F_r}, \quad (2e)$$

where $v_\varphi = u_\varphi - r/R_0$ is the azimuthal velocity in the reference frame corotating with the Sun. $R_0 = \bar{u}/(L\Omega)$ is the Rossby number ($\Omega$ is the equatorial angular rotation rate of the Sun). The six dependent variables for $(2a)$–$(2e)$ are the same as for $(1a)$–$(1f)$, but the system $(2a)$–$(2e)$ includes only five differential equations. The following condition is used to reduce the number of the equations. We assume that there is no steady electric field in the rotating frame (Pizzo, 1982), thus, the vectors of velocity and magnetic field are parallel: $B_\varphi/B_r = v_\varphi/u_r$, and we can calculate, for instance, the azimuthal component of magnetic field $B_\varphi$ from this relation.

2.2. Initial state

To solve the mixed initial-boundary value problem in region I, we use the approach applied by Endler (1971) and by SSW, which consists of searching for a steady-state solution as a limit of the time-dependent evolution of an initial state according to the governing system $(1a)$–$(1f)$. Hence, we have to prescribe some distribution for dependent variables initially. Although this distribution may be arbitrary to some extent, it determines the main features of the resulting state (e.g., its transonic character).
As an initial state for hydrodynamic variables, it is assumed that a spherically-symmetric polytropic flow is given by a Parker-type solution of the 1-D HD equations (Parker, 1963). The following reference values discussed by SSW were used at 1 $R_S$: proton density $N_0 = 2.25 \times 10^8$ cm$^{-3}$ and temperature $T_0 = 1.8 \times 10^6$ K. The polytropic index $\gamma$ is assumed to be equal to 1.05. The radial velocity $u_r$ at 1 $R_S$ (10 $R_S$) is 8.1 (266) km s$^{-1}$. For region II we assume for $\gamma$ the value of 1.17 that has been deduced from the solar wind observations by Sittler and Scudder (1980).

Endler (1971) and SWW both assumed a dipole magnetic field configuration for the initial state (and for boundary conditions as well). Our aim was to try to incorporate the solar magnetic field observations at the solar photosphere, so we have turned to the measurements at the J. Wilcox Solar Observatory of Stanford University (Hoeksema, 1984; Hoeksema and Scherrer, 1985). To demonstrate the technique outlined in the present study, we have chosen the solar magnetic data for Carrington rotation 1682 (Figure 1, Hoeksema and Scherrer, 1985). The rotation covers the time interval near the maximum phase of solar cycle 21 (May 23–June 19, 1979). The epoch of high solar activity, when the heliospheric current sheet (HCS) is strongly warped, has been chosen for analysis because of the two-dimensional formulation of the problem. The formulation immediately implies the neglect of meridional gradients; hence, the greater the HCS inclination to the equator, the better the 2-D approximation is worked out.

![Fig. 1. A map of constant radial field strength on the source surface (2.5 $R_S$) of a potential field model for Carrington rotation 1682 computed by Hoeksema and Scherrer (1985). The solid (dashed) lines indicate the field directed away from (toward) the Sun. The contours are at 0 (the bold line), 1, 2, 5, and 10 $\mu$T.](image)

To compute initial values for $B_r$ and $B_\phi$, we used the magnitudes of spherical harmonics (up to third order) calculated by Hoeksema and Scherrer (1985) who used a potential magnetic field model for a spherical shell between 1 $R_S$ and the ‘source surface’ at 2.5 $R_S$. On and outside the shell we assume that the field is purely radial and decreases as $r^{-2}$. The latter follows the divergence-free condition. Figure 2 shows the field line and velocity map for the initial state. The field lines were traced from the solar surface to 10 $R_S$ with a spacing of 10° in longitude. One can see that the six-sector
structure existing near the Sun transforms to a two-sector one at about $2.5 R_\odot$ (cf. Figure 1). The azimuthal distributions of the initially prescribed $B_r$, $B_\phi$ at $1 R_\odot$ are shown in Figure 3. The radial variation of the thermodynamic variables between 1 and $5 R_\odot$ is seen in Figure 3 of SSW.

2.3. Numerical method

The method employed to trace the temporal evolution of the initial state by advancing the solution from $t$ to $t + \Delta t$ for region I and to step the solution from $r$ to $r + \Delta r$ in region II is a highly efficient numerical scheme developed by MacCormack (1971) and applied to the solar wind flow simulation by Pizzo (1978, 1980, 1982). Note that this scheme competes successfully with the commonly used two-step Lax–Wendroff scheme (Peyret and Taylor, 1983). The MacCormack scheme is an explicit, two-step, second-order, predictor-corrector technique.
The two-dimensional grid, applied in region I to rewrite Equations (1a)–(1f) in finite-difference form, consists of 38 points in longitude with a spacing of 10° (36 routine points and two auxiliary ones to conveniently assign periodic boundary conditions). To reduce the number of grid points along the radius and at the same time to account for the most sharp gradients near the solar surface, the radial coordinate $r$ was transformed to a new logarithmic one, $r_1 = \ln(r/r_0)$, where the parameter $r_0$ is assumed to be equal to $1 R_S$. If the grid points are equally spaced along $r_1$, grid spacing along $r$ increases proportionally to $r(\Delta r = r\Delta r_1)$. Equations (1a)–(1f) were accordingly transformed to the new independent variable before applying the numerical marching scheme. The similar non-equidistant-radius grid has been used previously by Wang, Hu, and Wu (1982) and Suess (1982). To cover the interval between 1 and 10 $R_S$, only 25 points are needed if the grid spacing is 0.1 $R_S$ at 1 $R_S$. It increases up to about 1 $R_S$ at the top boundary.

The important computational point is that the system (1a)–(1f) is written in a mixed conservation (Equations (1a)–(1d)) and non-conservation (Equations (1e)–(1f)) form. The reason for such a selection is that the momentum equations (1e) and (1f) should be written in non-conservation form, as recommended by Brackbill and Barnes (1980), to partially prevent the appearance of non-physical forces along the field lines. It should be noted that small artificial forces always appear in numerical calculations, even if the boundary conditions are properly prescribed (see the paper by Yeh and Dryer, 1985) as a simple consequence of a finite-difference approximation of the governing equations. But if the momentum equation is written in a conservation form, the cumulative effect of the forces over many time steps can be crucial for the flow structure in regions of low Alfvén Mach number ($<1.0$) and the solution may be totally destroyed due to numerical violation of the $\nabla \cdot \mathbf{B} = 0$ law. The conservation form is maintained for the mass (1a), induction (1b), (1c), and energy (1d) equations. We believe that this procedure allows us to account for possible discontinuities in the solution (as current sheets). As for Equations (2a)–(2e), all of them are written in conservation form to describe the strong shocks which may appear in supersonic flow in region II.
According to the mixed form of the system (1a)–(1f), we applied the numerical MacCormack scheme to Equations (1a)–(1d) in conservation form and to Equations (1e)–(1f) in non-conservation form. The purely conservation form has been applied to Equations (2a)–(2e) for region II.

To maintain the numerical stability of computations in region I, we applied the commonly used (SSW; Han, 1977) artificial diffusion scheme developed by Lapidus (1967). For the smoothing constants in the scheme we use values of 4 (c_r = c_0 = 4) as was done by SSW. However, as it turned out, the explicit dissipation alone is not enough to maintain the computational stability (because of oscillations appearing in the regions of small B_φ). Thus, we were forced to impose an additional ad hoc restriction: the azimuthal field B_φ cannot change sign from that prescribed initially.

The necessary (but not sufficient) stability criterion for any explicit numerical scheme is to choose the time (or radial) step size from the Courant–Friedrichs–Lewy condition. Taking into account the limitations to the step size imposed by explicit diffusion in region I (Han, 1977), we may write the following restrictions:

**region I:**

\[
\Delta t \leq \min \left( \frac{\Delta r}{\max |\lambda_r|}, \frac{\Delta \varphi}{c_r \max |\partial u_r/\partial r|}, \frac{0.5}{c_0 \max |\partial u_\varphi/\partial \varphi|} \right),
\]

\[
\lambda_r = u_r \pm \left[ \frac{1}{2} (V_{a_r}^2 + C_s^2) + \left( \frac{1}{4} (V_{a_\varphi}^2 + C_s^2)^2 - C_s^2 V_{a_\varphi}^2 \right)^{1/2} \right],
\]

\[
\lambda_\varphi = \frac{1}{r} \left( u_\varphi \pm \left[ \frac{1}{2} (V_{a_\varphi}^2 + C_s^2) + \left( \frac{1}{4} (V_{a_r}^2 + C_s^2)^2 - C_s^2 V_{a_r}^2 \right)^{1/2} \right] \right);
\]

**region II:**

\[
\Delta r \leq \frac{\Delta \varphi}{\max |\lambda_\varphi|}, \quad \lambda_\varphi = \frac{u_\varphi^2 v_\varphi \pm \left[ (u_r^4 + \Delta) (v_{a_r}^2 u_r^2 - \Delta) \right]^{1/2}}{r \Delta},
\]

where \(\Delta = C_s^2 (u_r^2 - V_{a_r}^2) + u_\varphi^2 (V_{a_\varphi}^2 - u_\varphi^2), \quad V_a = (V_{a_r}^2 + V_{a_\varphi}^2)^{1/2}\) is the full Alfvén velocity, and \(C_s^2 = \gamma (P/\rho)\) is the square of the sonic velocity.

### 2.4. Boundary Conditions

A very important part of the problem formulation is to prescribe properly boundary values for the dependent variables at physical and computational boundaries in order to preserve numerical stability and physical consistency (e.g., Chen, 1973).

The only physical boundary in our case is the bottom radial limit for region I. (We will ignore the zero pressure boundary condition at \(r \to \infty\).) At the boundary (denoted as the inner boundary below) the method of projected characteristics can be applied to specify boundary conditions (Endler, 1971; Han, 1977; SSW; Hu and Wu, 1984). Since the flow is assumed to be subsonic, sub-Alfvénic and directed anti-sunward, there are two outgoing characteristics (coming from the region of interest to the boundary) and four incoming ones (coming from the boundary to the region of interest). Respectively, only four dependent variables can be prescribed arbitrarily and the other two have...
to be determined from differential equations (which are referred to as compatibility relations) along the outgoing characteristics. However, as shown by Steinolfson and Nakagawa (1976) and confirmed by SSW, application of the compatibility relations may be effectively replaced by an extrapolation procedure. Note that the neglect of the proper treatment of boundary conditions can result in failure to achieve a steady state. Washimi, Yoshino, and Ogino (1987) fixed all dependent variables at the inner boundary (prescribing, thus, permanent mass inflow through the solar surface and, in particular, in closed field regions); apparently, as a result, they could not obtain a steady-state solution. Linker, Van Hoven, and Schnack (1990), who used the same ‘fixed’ procedure, also obtained only a quasi-steady state.

We use the following combination of boundary conditions for the inner boundary. The radial magnetic field $B_r$ is assumed to be fixed at the initial values derived from the observations (Figure 3). In the course of the time-relaxation process the radial velocity $u_r$ at any boundary point is also fixed either at the initial value, if the flow at the adjacent (along $r$) point is directed anti-sunward, or at zero if the flow at the adjacent point turns backward toward the Sun. To prescribe the boundary values for the pressure, $P$, we use the following procedure. Although the radial velocity at the boundary is fixed ($u_r^c$), a value $u_r^e$ for radial velocity is calculated from the values at two grid points adjacent to the boundary using a linear extrapolation procedure. Maintaining the fixed value $u_r^c$ at the surface, we assume that the radial moment, which is approximated by using the difference $\Delta u_r = u_r^e - u_r^c$ as $\rho \Delta u_r^2/2$, is artificially converted to thermal energy (cf. Han, 1977) and, hence, the modified pressure $P'$ is given by

$$P' = P + (\gamma - 1) \rho \frac{\Delta u_r^2}{2} \left( - \frac{\Delta u_r}{\Delta u_r} \right).$$

The boundary values for the azimuthal field $B_\phi$ are linearly extrapolated from the values at two grid points adjacent to the boundary. The azimuthal speed $u_\phi$ is calculated from the condition that the vectors of the magnetic field and velocity are parallel at the inner boundary. The density $\rho$ is determined from the condition $P'/\rho = P_0/\rho_0$, where $P_0$ and $\rho_0$ are the references values of pressure and density, respectively, at the solar surface (SSW). This set of boundary conditions was actually used in the test example of coronal streamer simulation (Section 3). As to the simulation for CR 1682, we have chosen to fix the azimuthal magnetic field, $B_\phi$, at the solar surface instead of using the extrapolation procedure, since, as it turned out, this slightly inconsistent (from the projected characteristics method point of view) set of boundary conditions leads to a better agreement of the observed and calculated parameters at the Earth’s orbit. We emphasize that, in the present study, we do not impose any longitudinal variation of boundary values of hydrodynamic parameters in the initial state. Hence, the longitudinal effects, which will be described below, are exclusively caused by the influence of non-homogeneous magnetic field.

Both regions I and II have artificial computational boundaries at $\varphi = 0$ and $\varphi = 360^\circ$. Periodic boundary conditions are prescribed at the boundaries. Region II has no more boundaries, but region I has another computational boundary at the top radial limit.
(10 \( R_S \)). At this boundary, where the flow is assumed to be supersonic and super-Alfvénic and, hence, all the characteristics go from the region of interest to the boundary (outgoing characteristics), a linear extrapolation of all the conservation variables from grid points which are nearest to the boundary is used to assign extreme values to dependent variables.

### 3. Test Example

To assure physical and numerical validity of the considerations described above for region I, we have turned to the problem of coronal streamer formation with a model field of dipole type at the solar surface, which has been studied by SSW. The main difference with SSW is that our examinations are related to the equatorial plane, and the model dipole is assumed to be localized in that plane. The following formulae describe the field components of the dipole in the equatorial plane (Han, 1977):

\[
B_r = 2B_0 r^{-q} \cos \varphi, \quad B_\varphi = B_0 (\sqrt{5} - 1) r^{-q} \sin \varphi,
\]

where \( q = (\sqrt{5} + 3)/2 \) and \( B_0 \), the reference magnetic field at the solar surface, is taken as 2.35 G in the present example. Because of symmetry, only a part of our grid described in Section 2.3 is considered with a spacing in longitude of 2.5°. Symmetrical boundary conditions discussed by SSW at the 'pole' (\( \varphi = 0 \)) and the 'equator' (\( \varphi = 90^\circ \)) of the model dipole were assumed. As to the inner boundary, its treatment was the same as

![Diagram](Fig. 4a).

**Fig. 4a.** Test example of coronal streamer simulation in the solar equatorial plane: (a) the initial state, (b) the calculated steady-state solution for magnetic field and flow parameters, and (c) for pressure distribution. The 'pole' is the horizontal axis and the 'equator' is the vertical axis. The pressure is referred to the respective initial values. The peak values of the radial velocity and pressure are denoted by \( V_{max} \) and \( P_{max} \), respectively, and the contour intervals for pressure by \( \Delta \).
that described in Section 2.4. To prescribe the initial values to the hydrodynamic dependent variables we applied the same reference parameters for surface density and temperature as those used by SSW and specified in Section 2.2.

The steady-state magnetic field configuration, flow structure, and pressure distribution resulting from the temporal evolution of the initial dipole magnetic field superimposed on a spherically-symmetric flow are shown in Figure 4 in the same format as

Fig. 4b, c.
that used by SSW. Elapsed time for this final state is 32 hours. Note, however, that evolution of the solution is noticeable only up to times of about 16 hours. After that time the solution continues as a steady one as part of this time-relaxation technique.

Figures 5 and 6 shows the radial (at the ‘pole’ and at the ‘equator’) and the azimuthal (at 10 $R_S$) variations of the radial velocity, pressure, and temperature for the steady state. Comparing the figures with those displayed by SSW for $\beta = 0.5$, one can see that the main features are fairly well reproduced. The flow is aligned with the magnetic field and has a stagnation region with closed magnetic field structure near the equator. In this region magnetic forces are balanced by the pressure gradient, and at the top of this region, the flow accelerates and becomes super-Alfvénic. An important point inferred by SSW and confirmed in our calculations should be emphasized: at the top boundary the radial velocity has a maximum which is not centered on the open field-line region ($\phi = 0^\circ$) but, instead is shifted in the direction of the closed field region to longitudes of about 60°.

![Radial distribution](image)

Fig. 5. Test example of coronal streamer simulation in the solar equatorial plane: the radial distributions of the radial velocity, pressure, density, and temperature in the steady state at the ‘pole’ and at the ‘equator’ of the magnetic dipole. Note that the ‘pole’ and ‘equator’ are defined in the solar equatorial plane as $\phi = 0^\circ$ and $90^\circ$, respectively.

4. Numerical Results and Discussion

The steady state for the case of the equatorial plane during CR 1682 is shown in Figures 7–9. This state resulted from numerical simulation after 32 hours of relaxation from the initial state (Section 2.2) in region I. Figure 7(a) presents a planar map of
Fig. 6. Test example of coronal streamer simulation in the solar equatorial plane: the azimuthal variation at 10 $R_\odot$ of the radial velocity, pressure, density, and temperature in the steady state.

Fig. 7a. Results of simulation for CR 1682 in region I: the calculated steady-state solution for magnetic field and flow parameters (a), and for pressure (b).
magnetic field lines and flow velocities. The flows are aligned approximately along the magnetic field lines. (The slight deviations, which can be seen in Figure 7(a), are caused by minor inadequacy of an interpolation procedure used in tracing magnetic field lines, especially near current sheets.) The pressure map for the steady state is shown in Figure 7(b); three regions of higher pressure are shown to extend above the closed field regions in the flow field. Figure 8 shows the radial distributions of radial velocity and thermodynamic parameters at two selected longitudes. The first one ($\phi = 140^\circ$) is located across the closed field region and the second one ($\phi = 180^\circ$) in the open field region. The results are similar to those obtained for the coronal streamer simulation (Section 3). The azimuthal variations of the same parameters at 10 $R_S$ are shown in Figure 9. One can see that pressure and density curves are similar to each other (this is a consequence of the small value of the polytropic index, $\gamma = 1.05$) and both are anti-correlated with respect to the velocity curve.

We do not present the intermediate stages of the evolution since they have little physical meaning (Suess, 1983). However, the time the system takes to approach the steady state does have meaning as the characteristic time of relaxation of the solar corona with respect to large-amplitude perturbations (Suess, 1983). Our computations support a value of about 16–24 hours for the relaxation time in the coronal region up to 5–10 $R_S$ inferred from the studies by Endler (1971) and SSW.

The values of dependent variables at the top boundary of region I (10 $R_S$) were used as a boundary condition for the solution in region II up to the Earth’s orbit on a
Fig. 8. Results of simulation for CR 1682 in region I: the radial variation of the radial velocity, pressure, density, and temperature in the steady-state at \( \varphi = 180^\circ \) (open field region) and at \( \varphi = 140^\circ \) (closed field region) in the steady state.

Fig. 9. Results of simulation for CR 1682 in region I: the azimuthal variation of 10 \( R_S \) of the radial velocity, pressure, density, and temperature \( T \) in the steady state.
longitude grid with a spacing of $10^\circ$ (this takes only about 20 steps along the radial coordinate). Since solar rotation is taken into account, the interaction of solar wind streams of different velocities is self-consistently reproduced.

Figure 10 shows the results of the present simulation for Carrington rotation 1682 at the orbit of the Earth. It presents by dashed lines the calculated solar wind velocity $V$, the radial magnetic field $B_z$ ($B_x = -B_z$ if solar ecliptic (GSE) coordinates are assumed), the number density $N$, and the solar wind temperature $T$. The same parameters observed by spacecraft (King, 1986) are shown by the full line. One can see reasonably good agreement of general trends in the observed and computed velocity, radial magnetic field, density, and temperature. However, the computed variation in the solar wind velocity is about 30 km s$^{-1}$ instead of 300 km s$^{-1}$, as observed. This discrepancy seems to be a consequence of spherically symmetric, radially-independent heat addition which is implied by the polytropic approximation used with the non-adiabatic value of $\gamma$. The other result of the polytropic assumption is that the calculated numerical density is about an order of magnitude higher compared with the observed one.

![Figure 10](image-url)

Fig. 10. Results of simulation for CR 1682 at the Earth's orbit: the curves plotted as dashed lines are the calculated solar wind velocity $V$, the radial magnetic field $B_z$, the number density $N$, and the solar wind temperature $T$. The solar wind parameters observed by spacecraft are shown by the solid lines. Note the different scales for the observed and computed variables (except for the magnetic field).

Required development of the proposed technique concerns mostly the following aspects.

(a) Incorporation of explicit (decreasing with radius) heat addition terms in the energy and momentum equations as discussed, e.g., by Holzer (1977). This procedure would remove the polytropic approximation and account for a way to varying heat transfer from HD/MHD waves to the solar wind flow when the coronal magnetic field varies from an open to a closed configuration. This has to be done to adjust the
calculated number density to observations at the Earth's orbit and near the Sun at the same time, and to account for the observed level of modulation in the solar wind velocity during a solar rotation as in the example studied here.

(b) Upgrading the present two-dimensional formulation to a three-dimensional one. This is an important task if we want to account for epochs of low solar activity when the HCS is only slightly inclined with respect to the ecliptic plane, and meridional gradients are of vital importance. The only limitation for this generalization seems to be available computer facilities, but the fact of that we could obtain the results described above using a PC/AT 286 computer makes us sure of such a possibility. Note that a full cycle of computations takes about several hours of computer time.

(c) Incorporating the time-dependent processes of the CME flare streams' propagation in the solar atmosphere with the steady-state solution as a background, using observations to prescribe time-dependent boundary conditions and to test results of the simulation. (This has been done by Steinolfson (1982) for a model streamer-like configuration and by Steinolfson and Dryer (1984) and Panitchob (1987) in a 1-D formulation.)

The proposed numerical technique is thought to be of some value (especially being advanced according to the points specified above) for interpretation of solar-terrestrial relations and in predictions of geomagnetic activity, which is controlled by the solar wind, and the IMF parameters (Pudovkin et al., 1980; Usmanov, 1990). But these problems extend beyond the scope of the present paper and will be addressed in future studies.

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