A Global 3-D MHD Solar Wind Model with Alfvén Waves

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Abstract. A fully three-dimensional solar wind model that incorporates momentum and heat addition from Alfvén waves is developed. The proposed model upgrades the previous one [Usmanov, 1993a,b], by considering self-consistently the total system consisting of Alfvén waves propagating outward from the Sun and the mean polytropic solar wind flow. The simulation region extends from the coronal base (1 $R_\odot$) out to beyond 1 AU. The fully 3-D MHD equations written in spherical coordinates are solved in the frame of reference corotating with the Sun. At the inner boundary, the photospheric magnetic field observations are taken as boundary condition and wave energy density is prescribed to be proportional to the magnetic field strength. An example of the model application is presented.

Introduction

Unlike the previous version of solar wind model [Usmanov, 1993a,b], the model presented in this paper includes a non-homogeneous source of energy and momentum addition in the solar wind. This source is assumed to be different for the high-speed streams emanating from the regions with open magnetic field configuration (coronal holes) and for the low-speed solar wind. This non-homogeneous source is Alfvén waves.

Why Alfvén waves should be incorporated into the solar wind model? There is an extensive literature of theoretical works concerned with the Alfvén waves in the solar wind (see, e.g., the review by Holts et al. [1978]). The dominating point of view is that the Alfvén waves in the solar wind are undamped remnants of the wave flux (originating in the solar convective zone) that heats the solar chromosphere and lower corona. The existence of Alfvén waves in the solar wind and their importance for the energetics of solar corona and solar wind are known for a long time. Belcher et al. [1969] and Belcher and Davis [1971] detected Alfvén waves at 1 AU, and then Belcher [1971] and Alasriki and Couturier [1971] stated that Alfvén waves create an additional pressure and do work in solar wind acceleration.

MHD equations with Alfvén waves

The time-dependent equations for the total system consisting of Alfvén waves and the mean polytropic flow in the corotating with the Sun frame of reference are [Bretherton, 1970; Dewar, 1970]

\[
\begin{align*}
\frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, \\
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + 2(\hat{\Omega} \times \mathbf{v}) + \hat{\Omega} \times (\hat{\Omega} \times \mathbf{r}) &= -\frac{1}{\rho} \nabla p - \frac{GM_\odot}{r^2} \hat{r} + \frac{1}{4\pi \rho} \mathbf{rot} \mathbf{B} \times \mathbf{B} - \frac{1}{2r} \nabla \varepsilon, \\
\frac{\partial \mathbf{B}}{\partial t} &= \mathbf{rot} (\mathbf{v} \times \mathbf{B}),
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} \left[ \frac{1}{2}(v^2 - |\Omega \times r|^2) + \frac{P}{\gamma - 1} + \frac{B^2}{8\pi} - \frac{\rho GM_\odot}{r} + \varepsilon \right] &= -\nabla \cdot \left\{ \left[ \frac{\rho}{2}(v^2 - |\Omega \times r|^2) + \frac{\gamma P}{\gamma - 1} - \frac{\rho GM_\odot}{r} \right] \mathbf{v} + \frac{B}{4\pi} \times (\mathbf{v} \times \mathbf{B}) \left( \frac{3}{2} \mathbf{v} + \mathbf{V}_A \right) \varepsilon \right\}, \\
\frac{\partial \varepsilon}{\partial t} + \nabla \cdot [(\mathbf{v} + \mathbf{V}_A) \varepsilon] + \frac{\varepsilon}{2} \nabla \cdot \mathbf{v} &= 0,
\end{align*}
\]

where the dependent variables are $\rho$ the density, $\mathbf{v} = \mathbf{u} - \hat{\Omega} \times \mathbf{r}$ the velocity in corotating frame of reference ($\mathbf{u}$ is the velocity vector in inertial frame), $\mathbf{B}$ the magnetic field vector, $P$ the pressure, and $\varepsilon$ the Alfvén wave energy density. Other notations are the time $t$, the polytropic index $\gamma$, the universal gravitational constant $G$, the solar mass $M_\odot$, a unit vector in the radial direction $\hat{r}$, the angular velocity of solar rotation $\hat{\Omega}$, the Alfvén velocity $V_A = B/(4\pi \rho)^{1/2}$. Note that equations (1–5) are derived in the WKB approximation, i.e., variations of the background are small over times and distances comparable with the period and wavelength of the wave.

The steady-state equations in the rotating frame of reference with account the momentum and energy addition from the Alfvén waves are obtained from (1–5) by setting $\partial/\partial t = 0$ and replacing the induction equation for the magnetic field by the divergence-free equation. We do not assume the presence of stationary electric field in the rotating frame of reference, thus the velocity and magnetic field vectors are parallel in this frame [Pizzo, 1982]. The relation $\mathbf{v} || \mathbf{B}$ closes the system of the steady-state equations.
Numerical considerations

Computational domain

The computational domain extends from $1 \, R_\odot$ out to beyond $1 \,AU$ and consists of two parts: region I: $1 \leq r \leq 54.6 \, R_\odot$ and region II: $r \geq 54.6 \, R_\odot$. The steady-state solution in region I is sought for as a result of time relaxation process (forward integration in time of equations (1–5)), while the steady-state solution in region II is constructed by forward integration of the steady-state equations along $r$ [Pizzo, 1982]. Initial values of dependent variables at the base of region II $(54.6 \, R_\odot)$ are prescribed from the solution of the flow problem in region I.

Initial state

To formulate an initial 3-D distribution of dependent variables, we start from a solution of the 1-D problem that is a generalization of Parker's solution for inclusion of Alfvén waves propagating outward from the Sun. We consider one-dimensional polytropic solar wind flow with the wave energy and momentum addition from Alfvén waves. Under the assumption that the flow and the magnetic field are radial (hence, $B_r \sim r^{-2}$), the continuity, momentum, total energy, and wave energy equations are

$$r^2 \rho_u u_r = \Phi, \quad (6)$$

$$\rho u_r \frac{d u_r}{dr} + \frac{d}{dr} (P + \frac{\rho u^2}{2}) + \frac{\rho GM_\odot}{r^2} = 0, \quad (7)$$

$$r^2 \left[ u_r \left( \frac{\rho u^2}{2} + \frac{\gamma \rho P}{\gamma - 1} - \frac{\rho GM_\odot}{r} \right) + \frac{3}{2} u_r + V_A \right] \frac{d\rho \rho}{dr} = F_T, \quad (8)$$

$$\frac{d}{dr} \left( r^2 (u_r + V_A)^2 \rho \right) = 0, \quad (9)$$

where $\Phi$ and $F_T$ are mass and energy fluxes, respectively, per unit solid angle, $V_A = B_r / (4 \pi \rho)^{1/2}$, $B_r = B_0 (r_0 / r)^2$, and $B_0$ is the magnetic field strength at an initial level $r = r_0$. Equations (6–9) can be reduced to a single equation

$$u_r^2 - c_s^2 - u_r^2 \frac{d u_r}{dr} = 2 (c_s^2 + u_r^2) - \frac{GM_\odot}{r^2}, \quad (10)$$

where

$$c_s^2 = (\gamma - 1) \left[ \frac{F_T}{\Phi} - \frac{u_r^2}{2} + \frac{GM_\odot}{r} - \frac{3}{2} \frac{V_A}{u_r} \right],$$

$$u_a^2 = \frac{3 u_r + V_A}{4 (u_r + V_A) \rho} \frac{E}{\rho} = \frac{V_A r_0^2 (u_r^2 + V_A^2)^2}{V_A r_0^2 (u_r + V_A)^2} \frac{E_0}{\rho},$$

$$F_T = \Phi \left( \frac{1}{2} u_r^2 + \frac{\gamma}{\gamma - 1} \frac{2 k_B T_0}{m_p} - \frac{GM_\odot}{r_0} \right) + \frac{r_0^2}{2} \frac{3 u_r^2 + V_A^2}{\rho}, \quad \frac{E_0}{\rho} = \frac{u_r^2}{2} - \frac{GM_\odot}{r},$$

$$u_r^2, T_0, \rho, \frac{E_0}{\rho}, \text{and} \frac{V_A^2}{\rho} \text{are the velocity, temperature, wave energy density, and Alfvén velocity, respectively, at} \ r = r_0, \ k_B \text{is Boltzmann's constant, and} \ m_p \text{is the proton mass. We assume that the wave energy density at the coronal base} (r = r_0) \text{is determined by the following relation}$$

$$\frac{E_0}{\rho} = \eta p_0^{1/2} B_0,$$

where $p_0$ is the thermal pressure at $r = r_0$ and $\eta$ is a free parameter.

The results of numerical integration of equation (10) are presented in Table 1. In order to obtain these solutions, we assume $r_0 = 1 \,R_\odot$, the polytropic index $\gamma = 1.12$, the mass flux to be constant $\Phi = 1.5 \cdot 10^{11} \text{g s}^{-1} \text{cm}^{-2}$, temperature at the coronal base $T_0 = 1.8 \cdot 10^6 \text{K}$, and $\eta = 0.001$.

<table>
<thead>
<tr>
<th>$B_r$, Gauss</th>
<th>$u_r$, km s$^{-1}$</th>
<th>$\rho$, cm$^{-3}$</th>
<th>$E_0$, dyn cm$^{-2}$</th>
<th>$P$, dyn cm$^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $R_\odot$</td>
<td>4.6</td>
<td>4.0 \times 10^7</td>
<td>5.3 \times 10^{-4}</td>
<td>2.0 \times 10^{-2}</td>
</tr>
<tr>
<td>1 AU</td>
<td>701.4</td>
<td>5.7</td>
<td>2.1 \times 10^{-9}</td>
<td>4.3 \times 10^{-10}</td>
</tr>
<tr>
<td>2.0</td>
<td>9.3 \times 10^7</td>
<td>0</td>
<td>4.6 \times 10^{-2}</td>
<td>1.0 \times 10^{-9}</td>
</tr>
</tbody>
</table>

Table 1. Solutions of 1-D equations with Alfvén waves for two magnetic field strengths at the coronal base

To start the time relaxation process in region I, we assume at $t = 0$ a pure radial flow given from the solution of 1-D problem with the wave energy density at the coronal base varying according to the distribution of radial magnetic field at $1 \,R_\odot$ calculated from Hoeksema's expansion coefficients [Hoeksema and Scherrer, 1985] truncated at the 3rd order terms. The magnetic field at $t = 0$ is assumed to be purely radial too and to decrease as $r^{-2}$.

Numerical scheme

We introduce a composite mesh that consists of three overlapping spherical meshes. The first one is the usual spherical mesh with a limited extension in latitude $(42 \leq \theta_1 \leq 138^\circ)$, $0 \leq \phi_1 \leq 360^\circ$). The polar axis of the mesh is directed along the solar rotation axis. Two other meshes are introduced to cover the polar regions in both hemispheres. These meshes are fragments of spherical coordinates $(36 \leq \theta_2 \leq 144^\circ, 36 \leq \phi_2 \leq 144^\circ)$, and $216 \leq \phi_3 \leq 324^\circ$) with the polar axis lying in the equatorial plane of the first coordinate system $(\theta_1 = 90^\circ, \phi_1 = 90^\circ)$. 


The composite mesh, as one can see at each radial level from the point of view located above the north pole of the Sun, is shown in Figure 1. We use the angular mesh resolution of $12^\circ$ in both $\theta$- and $\phi$-direction for all the meshes. The first mesh contains 9 points along $\theta$- and 32 points along $\phi$-coordinate (two additional points at $\phi = -12$ and $360^\circ$ are inserted for convenience prescription of periodic condition at the boundary). Both north and south additional meshes consist of 10 points in $\theta$- and $\phi$-direction. The radial spacing is a linear function of $r$ and varies from 0.1 $R_\odot$ at $r = 1R_\odot$ to about 5 $R_\odot$ near the outflow boundary (54.6 $R_\odot$) and we need only 41 position along $r$ to cover all the region I. Thus, a total of $41 \times (9 \times 32 + 2 \times 10 \times 10) = 20,008$ grid points are considered in region I.

The finite-difference method employed in the present study to advance the solution along $t$ (region I) and along $r$ (region II) at interior mesh points is the numerical scheme by MacCormack [1971]. In order to maintain the numerical stability during the relaxation process in region I, the artificial diffusion scheme by Lapidus [1967] is applied (smoothing constants in each direction are chosen to be $c_r = 1$, $c_\theta = 8$, $c_\phi = 8$).

**Simulation Example**

In this section we present the results of the simulation for Carrington rotation 1682, which covers the time interval 23 May – 19 June 1979 near the maximum phase of solar cycle 21.

Starting from the initial state, the plasma-magnetic field system evolves in accordance with governing equations (1–5) and gradually approaches a steady state $\sim 48 - 64^h$ after the beginning of relaxation process in region I. The steady solution obtained for region I provides us with initial conditions for constructing a steady-state solution in region II. The polytropic index $\gamma$ in region II we chose to be equal to 1.47 (this value is inferred from observations by Totten et al., [1993]). Figure 2 demonstrates the results of simulation at 1 AU. The basic variables — the radial field, the radial velocity, and the number density — are presented as contour maps in Carrington coordinates. Figure 3 evaluates the correspondence of computed and spacecraft-observed [Cousens and King, 1986] parameters.

**Figure 1.** Composite mesh at each radial level from the point of view located above the north pole of the Sun. The second mesh covering the north pole region is embedded in the first one.

**Boundary conditions**

The only physical boundary for the problem under consideration is the inflow boundary of region I. On this boundary, the radial magnetic field $B_r$ is fixed at the initial values inferred from observations. The non-radial magnetic field components, $B_\theta$ and $B_\phi$, are assumed to be equal to zero at $t = 0$, but then linearly increase up to the values inferred from observations during 2 hours. After $t = 2^h$, all magnetic field components are kept constant at the boundary. The boundary value of wave energy density is calculated from the relation $\xi = \eta P^{1/2}B_r$, while the boundary conditions for other dependent variables are treated as discussed in [Usmanov, 1993a].

**Figure 2.** Results of simulation for Carrington Rotation 1682. Contour maps of constant radial velocity (in km s$^{-1}$) (a), number density (in cm$^{-3}$) (b), and radial field (in nT) (c) at 1 AU.

Acknowledgments. The research described in this paper was made possible in part by Grant No. R61000 from the International Science Foundation, by Grant No. R61300 from the International Science Foundation and Russian Government, by Grant No. 93-05-9082 from the Russian Foundation for Basic Research, and by support from the SOLMAG project. I wish to thank also the Austrian Academy of Sciences and the Space Research Institute for financial support during my exchange visits.

References