THREE-DIMENSIONAL MAGNETOHYDRODYNAMIC MODELING OF THE SOLAR WIND INCLUDING PICKUP PROTONS AND TURBULENCE TRANSPORT

Arcadi V. Usmanov1,2, Melvyn L. Goldstein2, and William H. Matthaeus1
1 Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA; arcadi.usmanov@nasa.gov
2 Code 673, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA
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ABSTRACT

To study the effects of interstellar pickup protons and turbulence on the structure and dynamics of the solar wind, we have developed a fully three-dimensional magnetohydrodynamic solar wind model that treats interstellar pickup protons as a separate fluid and incorporates the transport of turbulence and turbulent heating. The governing system of equations combines the mean-field equations for the solar wind plasma, magnetic field, and pickup protons and the turbulence transport equations for the turbulent energy, normalized cross-helicity, and correlation length. The model equations account for photoionization of interstellar hydrogen atoms and their charge exchange with solar wind protons, energy transfer from pickup protons to solar wind protons, and plasma heating by turbulent dissipation. Separate mass and energy equations are used for the solar wind and pickup protons, though a single momentum equation is employed under the assumption that the pickup protons are comoving with the solar wind protons. We compute the global structure of the solar wind plasma, magnetic field, and turbulence in the region from 0.3 to 100 AU for a source magnetic dipole on the Sun tilted by 0–90° and compare our results with Voyager 2 observations. The results computed with and without pickup protons are superposed to evaluate quantitatively the deceleration and heating effects of pickup protons, the overall compression of the magnetic field in the outer heliosphere caused by deceleration, and the weakening of corotating interaction regions by the thermal pressure of pickup protons.

Key words: magnetic fields – magnetohydrodynamics (MHD) – methods: numerical – solar wind – turbulence

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1. INTRODUCTION

The Sun is currently traveling through the local interstellar cloud that consists primarily of hydrogen with the number density of ~0.2 atoms cm−3 (Frisch et al. 2011). As a result of the Sun’s motion, the interstellar hydrogen enters the heliosphere and is ionized by solar EUV radiation and by charge exchange with solar wind protons. The interplanetary magnetic field picks up the newborn protons and entrains them into the solar wind (Axford et al. 1963; Parker 1969; Blum & Fahr 1970). The photoionization adds mass to the flow, while the charge exchange transfers momentum from solar wind protons to the newborn hydrogen atoms. The mass loading and the momentum loss reduce the bulk energy per particle and lead to a deceleration of the solar wind flow (Semar 1970; Holzer 1972). Because the initial speed of the pickup proton gyration about the local magnetic field is of order of the solar wind speed (in the frame of reference moving with the solar wind plasma), the pickup protons constitute a distinct population in the solar wind with an effective temperature that is much higher than that of the solar wind protons (Vasyliunas & Siscoe 1976; Isenberg 1986). Although the number density of pickup protons is significantly lower than that of solar wind protons, the higher temperature of pickup protons makes their thermal pressure a dominant component in the distant heliosphere (Burlaga et al. 1994, 1996).

The interstellar pickup protons are suprathermal particles coupled to the thermal interplanetary plasma through waves. The velocities of the newborn pickup protons initially fall in a narrow range of pitch angles to form a “ring” distribution. This distribution is unstable to a collective instability (Wu & Davidson 1972; Lee & Ip 1987). Note, however, that waves resulting from the instability have been only rarely observed, e.g., Joyce et al. 2010, 2012.) The subsequent wave–particle interaction leads to pitch-angle scattering that tends to isotropize the distribution. A fraction of the generated wave energy is presumably absorbed by solar wind protons. The heating associated with the pickup protons is believed to be the primary source of heat supplied to the solar wind in the outer heliosphere (e.g., Williams et al. 1995).

Another important effect of pickup protons is that they act to weaken corotating interaction regions (CIRs). This is due to two factors, one of which is that deceleration of the wind reduces the speeds of the interacting flows and, consequently, the degree of compression. The second factor is that the additional pressure of pickup protons lowers the Mach number, making the solar wind plasma less compressible. There is yet another effect of pickup protons that is also associated with the wind deceleration, viz., the deceleration causes an overall compression of the interplanetary magnetic field (Zank 1999) that is primarily azimuthal in the distant heliosphere. This compression does not appear to affect the density of solar wind protons because it is approximately balanced by the conversion of solar wind protons into pickup protons by the charge exchange (Usmanov & Goldstein 2006).

The role of pickup protons in the dynamics of the solar wind in the outer heliosphere has been a subject of quantitative studies with one-dimensional (radial) models since the early 1970s (e.g., Semar 1970; Holzer 1972; Isenberg 1986; Whang 1998; Wang et al. 2000, 2003; Wang & Richardson 2001). There are also a number of two- and three-dimensional models of the outer heliosphere that incorporate effects of pickup protons (e.g., Pauls et al. 1995; Zank et al. 1996; Linde et al. 1998; Opher et al. 2003; Pogorelov et al. 2009). However, the focus of the latter studies is on the interaction of the solar wind...
with the local interstellar medium rather than on properties of the solar wind itself. Also, those studies do not distinguish between pickup and solar wind protons because they assume that the pickup protons are thermally assimilated into the solar wind.

Isenberg (1986) noted that the interstellar pickup protons do not assimilate easily with the solar wind protons and suggested that the two species can be treated as separate proton fluids. Isenberg’s approach was further worked out by Fuh & Ziemkiewicz (1988), Whang et al. (1995, 2003), Whang (1998, 2010), Wang & Richardson (2001), Usmanov & Goldstein (2006), and Detman et al. (2011). Usmanov & Goldstein (2006) developed a three-dimensional solar wind model that incorporated the effects of pickup protons as separate fluid beyond 1 AU. Following Whang (1998), the solar wind protons and electrons were assumed to constitute a single fluid, while the pickup protons were considered as a second fluid described by separate continuity and energy equations. Both fluids were assumed to move with the same velocity. To specify an inner boundary condition at 1 AU, the output from the steady-state solar wind model of Usmanov & Goldstein (2003) was used. The simulation results of Usmanov & Goldstein (2006) are consistent with earlier studies (e.g., Whang 1998; Wang & Richardson 2001) in showing that the pickup protons cause a significant deceleration of the solar wind beyond 5–10 AU and an increase in average plasma temperature with heliocentric distance. The results are also in reasonable agreement with the estimates of pickup proton density, temperature, and thermal pressure derived by Burlaga et al. (1994, 1996) from observations of pressure-balanced structures on Voyager 2.

The three-dimensional character of the model of Usmanov & Goldstein (2006) allowed the authors to obtain an estimate of the latitudinal effect contained in the Voyager 2 data. Due to the temperature increase with latitude from the equatorial plane during solar minimum, the gradual descent of the spacecraft to southern latitudes (following its encounter with Neptune at \( \approx 30 \) AU) appears to have been the cause of the flattening of the radial profile of proton temperature. The model showed a good fit to the proton temperature measurements if the solar wind protons acquired \( \sim 1\% \) of thermal energy of pickup protons (in addition to the heating implied by the non-adiabatic index \( \gamma = 1.46 \) used by Usmanov & Goldstein 2006). However, the model did not contain any mechanism that accounted for energy transfer from pickup protons to solar wind protons.

An alternative approach to modeling the temperature distribution in the solar wind is based on the turbulence transport model that includes effects of the cascade and dissipation of magnetohydrodynamic (MHD) turbulence as well as pickup proton effects (Williams et al. 1995; Matthaeus et al. 1999, 2004; Smith et al. 2001, 2006; Isenberg et al. 2003; Breech et al. 2008, 2009). The model assumes that the waves generated by newborn pickup protons are a source of turbulence energy and therefore enhance the turbulent heating of the plasma. Based on the physical mechanism of plasma heating by turbulent dissipation, the model provides a framework for modeling the temperature distributions in the solar wind. The turbulence transport model is, however, one-dimensional (with only radial derivatives present in the governing equations) and assumes a constant wind speed, while the plasma density and magnetic field magnitude are predefined. These approximations obviously cannot deal in a complete way with non-radial effects, such as those demonstrated by Usmanov & Goldstein (2006), and with the solar wind deceleration that should affect the proton temperature distribution. Note that Isenberg et al. (2010) relaxed the constant-speed assumption and extended the turbulence transport model to include a linearly decreasing wind speed.

Recently, Usmanov et al. (2011) have generalized the turbulence transport model to include a simultaneously computed background solar wind and developed an axially symmetric solar wind model that describes properties of both the mean-field (ambient) solar wind and turbulence throughout the heliosphere from 0.3 AU to 100 AU. The model is based on the Reynolds decomposition of the physical variables into the average and fluctuating parts and the numerical solutions of the Reynolds-averaged solar wind equations coupled with a set of turbulence transport equations, similar to those used by Breech et al. (2008), for the turbulence energy, the normalized cross-helicity, and the correlation length. The model was used to study the interaction between the mean-field solar wind and turbulence and the role of turbulence in the temperature distribution in the solar wind. However, the pickup protons were not treated as separate fluid, and the model did not account for the solar wind deceleration. The effect of pickup protons was introduced only via a source term in the equation for turbulent energy and could affect only plasma temperature (indirectly through turbulent heating).

In this paper, we make another step forward to merge the two approaches to solar wind modeling—with pickup protons as a separate fluid (Usmanov & Goldstein 2006) and with the turbulence transport and heating (Usmanov et al. 2011)—by combining their advantages into an integrated fully three-dimensional model. While both approaches used the reference frame corotating with the Sun for constructing steady-state solutions for the solar wind, technically they are different. In the first approach, steady-state governing equations with pickup protons were integrated using a marching-along-radius numerical algorithm (e.g., Pizzo 1982; Usmanov 1993). On the other hand, the modeling with turbulent transport applied the method of time relaxation (e.g., Endler 1971; Steinolfson et al. 1982; Usmanov 1993), which requires integrating the time-dependent equations until a steady state is achieved. In this study, we will follow the second approach and employ time relaxation to obtain steady solutions of time-dependent equations. We first describe in detail the model formulation and discuss the initial and boundary conditions in Section 2. The results of the simulation and comparison with Voyager 2 data are presented in Section 3. We conclude with a summary of the results and a discussion of limitations and future extensions of the model in Section 4.

2. MODEL FORMULATION

2.1. Mean-field (Reynolds-averaged) Equations

Our model is “two-fluid” in the sense that we treat interplanetary pickup protons as one fluid and the solar wind protons and electrons as a second fluid. The two fluids are described by separate mass and energy equations and a single momentum equation. Following Whang et al. (1995, 2003) and Usmanov & Goldstein (2006), we write the governing time-dependent MHD equations in a frame of reference corotating with the Sun as

\[
\frac{\partial \tilde{N}_s}{\partial t} + \nabla \cdot (\tilde{N}_s \tilde{v}) = -q_{ex},
\]  (1)
\begin{align}
\frac{\partial \tilde{v}}{\partial t} + (\tilde{v} \cdot \nabla) \tilde{v} + \frac{1}{\tilde{\rho}} \nabla (\tilde{P} + \tilde{P}_t) - \left(\frac{\nabla \times \tilde{B}}{4\pi \tilde{\rho}}\right) \times \frac{GM_\odot}{r^2} \tilde{r} \\
\quad + 2\Omega \times \tilde{v} + \Omega \times (\Omega \times \tilde{r}) = - \frac{1}{\tilde{\rho}} (q_{\text{ex}} + q_{\text{ph}}) m_p u, \tag{2}
\end{align}

\begin{align}
\frac{\partial \tilde{B}}{\partial t} = \nabla \times (\tilde{v} \times \tilde{B}), \tag{3}
\end{align}

\begin{align}
\frac{\partial \tilde{P}}{\partial t} + (\tilde{v} \cdot \nabla) \tilde{P} + \frac{5}{3} \tilde{P}_t \nabla \cdot \tilde{v} = (q_{\text{ex}} + q_{\text{ph}}) \frac{m_p u^2}{3} - Q_{\text{Ps}}, \tag{4}
\end{align}

\begin{align}
\frac{\partial \tilde{N}_I}{\partial t} + \nabla \cdot (\tilde{N}_I \tilde{v}) = q_{\text{ex}} + q_{\text{ph}}, \tag{5}
\end{align}

\begin{align}
\frac{\partial \tilde{P}_I}{\partial t} + (\tilde{v} \cdot \nabla) \tilde{P}_I + \frac{5}{3} \tilde{P}_t \nabla \cdot \tilde{v} = (q_{\text{ex}} + q_{\text{ph}}) \frac{m_p u^2}{3} - Q_{\text{Pt}}(\tilde{r}). \tag{6}
\end{align}

where the independent variables are the time $t$ and the heliocentric position vector $\tilde{r}$. The dependent variables are the velocity in the corotating frame $\tilde{v}$, the magnetic field $\tilde{B}$, the number density of solar wind protons $N_\Lambda$, the combined pressure of solar wind protons and electrons $\tilde{P} = \tilde{P}_H + \tilde{P}_e$, the number density $\tilde{N}_I$, and thermal pressure $\tilde{P}_t$ of pickup protons. Since we will neglect pickup protons, $\tilde{P}_t$ can be simply replaced by $\tilde{P}_e$.

The sources of energy deposition/extraction are assumed to be due to turbulent dissipation $Q_{\text{Ps}}(\tilde{r})$ and wave generation $Q_{\text{Pt}}(\tilde{r})$ and are defined below. The production rates of pickup protons (per unit volume per unit time) from interstellar hydrogen atoms by charge exchange and photoionization are, respectively,

\begin{align}
q_{\text{ex}} = \sigma N_S N_{\text{H0}} u, \quad q_{\text{ph}} = v_0 \left(\frac{r_0^2}{r}\right)^2 N_I, \tag{7}
\end{align}

where $N_S$ is the number density of the interstellar hydrogen, $\sigma = 2 \times 10^{-15} \text{ cm}^2$ is the mean charge exchange cross section of hydrogen atoms, and $v_0 = 0.9 \times 10^{-7} \text{ s}^{-1}$ is the photoionization rate per hydrogen atom at the heliocentric distance $r_0 = 1 \text{ AU}$ (Whang 1998). The density of the interstellar hydrogen is assumed to increase with heliocentric distance as

\begin{align}
N_I = N_{\text{H0}} \exp \left(-\frac{L_{\text{cv}}}{r}\right),
\end{align}

where $N_{\text{H0}}$ is the number density of interstellar hydrogen at the termination shock and $L_{\text{cv}}$ is the characteristic radius of the ionization cavity around the Sun (Breech et al. 2008). Note that $N_I$ is independent of latitude and longitude and, as a result, the preferred direction of the interstellar neutral hydrogen inflow into the heliosphere is not taken into account in the present study.

Equations (1) and (5) are the continuity equations for the solar wind protons and pickup protons, respectively. The photoionization of the interstellar hydrogen does not change the density of solar wind protons, $\tilde{N}_S$, while the charge exchange process preserves the total proton density, $\tilde{N}_S + \tilde{N}_I$. The momentum Equation (2) is in a non-conservation form with the sink term on the right-hand side responsible for the momentum loss (Whang et al. 1995). Since this equation is written in the rotating frame, it contains the Coriolis and centrifugal forces (last two terms on the left-hand side). The induction equation (Equation (3)) has its traditional form. The energy equation (Equation (4)) for solar wind protons/electrons assumes that in the absence of a heat flux and energy sources, the adiabatic relation between $\tilde{P}$ and $\tilde{N}_S$ is valid (see Whang et al. 1995). The energy equation for pickup protons (Equation (6)) was derived by Whang et al. (1995) as the second moment of the Boltzmann equation for interstellar pickup protons (also see Whang 2010). Unlike the steady-state formulation employed by Usmanov & Goldstein (2006), Equations (1)–(6) are time dependent and describe the behavior of the solar wind plasma and interplanetary magnetic field at all (MHD) scales, including the large-scale solar wind structures and the fluctuations in solar wind parameters with periods from minutes to several hours.

As the brute-force approach to resolve all the dynamically relevant scales by direct numerical simulations (DNSs) is obviously impractical when turbulence is present, it is natural to turn to the Reynolds averaging method, in which the averages of physical quantities describe the mean-field (background) solar wind and the fluctuations being treated statistically. In applying this method, we will follow the approach of Usmanov et al. (2011) by considering a field of incompressible turbulence, superimposed on a fully compressible non-homogeneous background solar wind. Our present approach differs from that of Usmanov et al. (2011) in that the governing system of equations now includes two additional variables (pickup proton density and pressure) and two additional equations for the variables. We note here that our method based on the assumption of local incompressibility of fluctuations and the scale separated transport theory of Zhou & Matthaeus (1990) is similar to the two-timescale method suggested by Whang (1990).

We now decompose all physical variables into mean and fluctuating components $\tilde{a} = a + a'$, where $a$ is any of the physical variables, $\bar{a}$ is the mean, and $a'$ is a deviation from this mean. We assume that the means are averages over an infinite ensemble of realizations with the averages of the deviations being, by definition, zero, i.e., $\langle a' \rangle = 0$, while $\langle \ldots \rangle$ denotes the Reynolds ensemble averaging. Following the approach of Usmanov et al. (2011), we assume that the turbulence is locally incompressible and neglect the fluctuations of the solar wind proton and pickup proton density ($N'_S = 0$ and $N'_I = 0$). By substituting the decomposed quantities into Equations (1)–(6) and requiring that averaging operators meet the Reynolds conditions (Monin & Yaglom 1971), we obtain the following mean-field equations:

\begin{align}
\frac{\partial \tilde{N}_S}{\partial t} + \nabla \cdot (\tilde{N}_S \tilde{v}) = -q_{\text{ex}}, \tag{8}
\end{align}

\begin{align}
\frac{\partial (\rho v)}{\partial t} + \nabla \cdot \left[ (\rho vv - \frac{1}{4\pi} BB) + \left( P + P_t + \frac{(B^2)}{8\pi} + \frac{B^2}{8\pi} \right) I \right] \\
\quad + \left( \frac{G M_\odot}{r^2} \tilde{r} + 2\Omega \times \tilde{v} + \Omega \times (\Omega \times \tilde{r}) \right) \\
= -m_p (q_{\text{ex}} u + q_{\text{ph}} \Omega \times \tilde{r}), \tag{9}
\end{align}
\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \mathbf{e}_m),
\]  
(10)

\[
\frac{\partial P}{\partial t} + (\mathbf{v} \cdot \nabla)P + \gamma P \nabla \cdot \mathbf{v} + \frac{\gamma P}{N_S} q_{ex} + \nabla \cdot (\gamma P' \mathbf{v}') + (\gamma - 1)(P' \nabla \cdot \mathbf{v}') = -(\gamma - 1)\nabla \cdot \mathbf{q}_U + Q_1(\mathbf{r}),
\]  
(11)

\[
\frac{\partial N_I}{\partial t} + \nabla \cdot (N_I \mathbf{v}) = q_{ex} + q_{ph},
\]  
(12)

\[
\frac{\partial P_I}{\partial t} + (\mathbf{v} \cdot \nabla)P_I + \frac{5}{3} P_I \nabla \cdot \mathbf{v} + \nabla \cdot (\gamma P' \mathbf{v}') + \frac{2}{3} (P' I \nabla \cdot \mathbf{v}') = (q_{ex} + q_{ph}) \frac{m_p}{3} (u^2 + v^2) - Q_2(\mathbf{r}),
\]  
(13)

where the momentum equation (Equation (9)) is transformed into a conservation form, and \(\mathbf{e}_m = -(\mathbf{v}' \times \mathbf{B}')\) is the fluctuating electric field. The momentum equation (Equation (9)), the induction equation (Equation (10)), and the energy equation for pickup protons (Equation (13)) contain now the fluctuation-based terms that will need to be modeled separately.

### 2.2. Turbulence Equations

To obtain the evolution equation for incompressible fluctuations in the presence of pickup protons, we subtract Equations (9) and (10) from Equations (2) and (3), respectively, and use Equations (8) and (12) (see Appendix A):

\[
\frac{\partial \mathbf{z}^\pm}{\partial t} + [\mathbf{V}_A \cdot \nabla] \mathbf{z}^\pm + (\mathbf{z}^\pm \cdot \nabla)(u \pm \mathbf{V}_A) \\
+ \frac{\mathbf{z}^\pm - \mathbf{z}^\mp}{2} \left[ \nabla \cdot \left( \frac{u \pm \mathbf{V}_A}{2} \right) + \frac{q_{ph} m_p}{2 \rho} \right] + \mathbf{V}_A \frac{\mathbf{z}^\mp}{2 \rho} \cdot \nabla \rho \\
+ \frac{1}{\rho} \nabla P_T^\pm + \Omega \times \mathbf{z}^\pm + \frac{m_p}{2 \rho} (q_{ex} + q_{ph}) (\mathbf{z}^\pm + \mathbf{S}^\pm) \\
= N_L \mathbf{z}^\pm + \mathbf{S}^\pm,
\]  
(14)

where \(\mathbf{z}^\pm = \mathbf{v} \pm \mathbf{B}'/\sqrt{4\pi \rho}\) are the Ešlăsser variables (Ešlăsser 1950), \(\mathbf{V}_A = \mathbf{B}/\sqrt{4\pi \rho}\) is the Alfvén velocity, \(P_T^\pm\) is the fluctuation of the total thermal and magnetic pressures, \(N_L \mathbf{z}^\pm\) consolidate nonlinear terms, and \(\mathbf{S}^\pm\) are introduced to represent source terms that may or may not be present. This equation differs from Equation (9) in Usmanov et al. (2011) by the presence of extra terms that appear due to the source terms on the right-hand side of the continuity equations (Equations (1) and (5)). If the flow is super-Alfvénic and the fluctuations are both transverse and axisymmetric with respect to \(\mathbf{B}\), then the following transport equations can be obtained using the additional assumptions discussed by Breech et al. (2008) (see Appendix A in Usmanov et al. 2011 for details in the case without pickup protons):

\[
\frac{\partial Z^2}{\partial t} + (\mathbf{v} \cdot \nabla)Z^2 + \frac{Z^2}{2} \left[ \nabla \cdot \mathbf{u} + \frac{(2 q_{ex} + 3 q_{ph}) m_p}{\rho} \right] \\
+ \sigma_D Z^2 \left[ \nabla \cdot \mathbf{u} - \hat{\mathbf{B}} \cdot (\hat{\mathbf{B}} \cdot \nabla) \mathbf{u} + \frac{(2 q_{ex} + q_{ph}) m_p}{2 \rho} \right] \\
= -\frac{\alpha f^+(\sigma_c) Z^3}{\lambda} + \dot{E}_{pl},
\]  
(15)

where \(\dot{E}_{pl}\) is the source term for pickup proton generated turbulence (Williams et al. 1995; Isenberg et al. 2003; Smith et al. 2004, 2006; Breech et al. 2008).

In our notation

\[
\dot{E}_{pl} = f_D d N_I u \mathbf{V}_A \frac{\partial \rho}{\partial N_S} = f_D u \mathbf{V}_A (q_{ex} + q_{ph}),
\]  
(18)

where \(f_D\) is a parameter that accounts for the details of the isotropization process of pickup protons. Following Breech et al. (2008), we treat \(f_D\) as a constant, although in general it should depend on the kinetic details of the interaction between protons and waves (Isenberg et al. 2003; Isenberg 2005; see also the discussion in Breech et al. 2008).

We assume that solar wind heating comes from the turbulent cascade, pickup protons, and electron heat flux (approximated by the “closureless” Hallwog formula). The first term on the right-hand side of Equation (15) is the rate of deposition of turbulence energy into the internal energy of the solar wind plasma, and the second term is the turbulent energy supplied by the pickup protons. Consequently, the source terms in Equations (11) and (13) take the form

\[
Q_1(\mathbf{r}) = (\gamma - 1) \alpha f^+(\sigma_c) \rho Z^3, \\
Q_2(\mathbf{r}) = \frac{\rho \dot{E}_{pl}}{3}.
\]  
(19)

We further assume that the solar wind protons and electrons exert the same thermal pressure \(P_S = P_E = P/2\). In combination with the requirement of charge neutrality \(N_E = N_S + N_I\), where \(N_E\) is the electron number density, the proton and electron temperatures \(T_S = P/2k_B N_S\) and \(T_E = P/2k_B (N_S + N_I)\), where \(k_B\) is Boltzmann’s constant.

### 2.3. Turbulence-based Terms in the Mean-field Equations

The terms containing primed quantities in Equations (9), (10), and (13) must be modeled separately. With the assumptions that fluctuations are locally incompressible (\(\rho' = 0\)) and transverse and axisymmetric about \(\mathbf{B}\), the Reynolds stress tensor \((\rho \mathbf{v} \mathbf{v}' - \mathbf{B} \mathbf{B}' / 4\pi\)) can be written as \(\sigma_{D,0} Z^2 (1 - \mathbf{B} \mathbf{B}) / 2\). In terms of the turbulent energy and the normalized energy difference, the mean magnetic pressure due to the fluctuations in Equation (9) is \(B'^2 / 8\pi = (1 - \sigma_D) Z^2 / 4\). In this study, we continue to neglect the turbulent electric field (\(\mathbf{e}_m = 0\)) and the turbulent fluctuations of thermal pressures \((P' = P_E' = P')\). As a result, the mean with the term turbulent ram pressures \((\mathbf{v}' \mathbf{P}')\) and the dilatation term \((\gamma - 1) / (\mathbf{v}' \cdot \mathbf{v}')\) will disappear from Equations (11) and (13).
2.4. The Integrated System of Equations

To solve for the mean-field and turbulence variables simultaneously, we combine the mean-field Equations (8)–(13) and (15)–(17) into an integrated system of quasi-conservative equations. We introduce the following non-dimensional parameters: the Strouhal number $S_h = u_0 T_0 / L_0$, the Rossby number $R_o = \Omega L_0 / u_0$, the Euler number $E_u = P_0 / \rho u_0^2$, the Fréaud number $F_r = u_0^2 L_0 / G M_\odot$, and the Alfvén Mach number $M_A = u_0 (4 \pi \rho_0)^{1/2} / B_0$, where $L_0$, $T_0$, $\rho_0$, $u_0$, $B_0$, and $P_0$ are units of length, time, density, velocity, magnetic field, and pressure, respectively. The coupled system of Equations (8)–(13) and (15)–(17) can be then rewritten in the following non-dimensional quasi-conservation vector form:

$$\frac{\partial \mathbf{W}}{\partial t} + \nabla \cdot \mathbf{F} = \mathbf{S},$$

where $\mathbf{W}$, $\mathbf{F}$, and $\mathbf{S}$ are the vectors given by

$$(N_S, \rho \mathbf{u}, B, P^{1/\gamma}, N_i, P_I^{3/5}, Z^2 \sigma_c, \rho \lambda, E)$$

$$\mathbf{F} = S_h \left( \begin{array}{c} N_S \mathbf{v} \\ \rho (\mathbf{v} - \eta \mathbf{V}_A + \tilde{P}) I \\ \mathbf{v} B \\ \rho^{1/\gamma} (\gamma - 1) (\gamma - 1) \\ Z^2 \mathbf{v} \\ Z^2 \sigma_c \mathbf{v} \\ \rho \lambda \mathbf{v} \\ \mathbf{v} E + \mathbf{u} \tilde{P} - \rho \eta \mathbf{V}_A (\mathbf{u} \cdot \mathbf{V}_A) + q_{\eta I} \end{array} \right),$$

$$\mathbf{S} = S_h \left( \begin{array}{c} -\dot{q}_{ex} \\ -\dot{q}_{ex} \mathbf{u} - \rho \left( \frac{1}{F_r r^2} \mathbf{r} + R_o \hat{\Omega} \times \mathbf{u} \right) \\ 0 \\ -\frac{P^{1/\gamma}}{\gamma_1} - \frac{\gamma_1}{N_S} \left( \gamma - 1 \right) (\gamma + 1) \mathbf{f}^* (\sigma_c) \mathbf{Z}^3 \\ \frac{1}{5 P^{1/2} E_u} \left\{ \dot{q}_{ex} + \dot{q}_{ph} \right\} \left[ u^2 + (1 + \sigma_D) Z^2 \right] - \rho \mathbf{E}_P \\ \frac{Z^2}{2} \left\{ (1 - \sigma_D) \mathbf{V} \cdot \mathbf{u} + 2 \sigma_D \mathbf{B} \cdot \mathbf{V} \mathbf{u} \right\} \\ -\frac{1}{\rho} \left[ 2 \dot{q}_{ex} (\sigma_D + 1) + \dot{q}_{ph} (\sigma_D + 3) \right] \\ -\alpha f^* (\sigma_c) Z^3 \\ \frac{\lambda}{\rho} \left[ f^* (\sigma_c) Z^3 - \frac{\lambda \mathbf{E}_P}{\alpha Z^2} \right] + \dot{q}_{ph} \lambda \\ \rho \beta \left[ f^* (\sigma_c) Z^3 - \frac{\lambda \mathbf{E}_P}{\alpha Z^2} \right] + \dot{q}_{ph} \lambda \\ -\dot{q}_{ph} \frac{1}{F_r r} - q_{ex} \end{array} \right)$$

where $\tilde{P} = (P + P_I) E_u + (1 + \sigma_D) \rho Z^2 + \rho V_A^2 / 2$ is the total pressure, $\mathbf{V}_A = \mathbf{B} / (M_A \sqrt{\rho})$ is the non-dimensional Alfvén velocity, $\hat{\Omega}$ is the unit vector in the direction of $\mathbf{\Omega}$, and $q_{ex} = N_S N_i u_k K_1$ and $q_{ph} = N_I u_0^2 K_2 / r^2$ are the non-dimensional production rates of pickup protons, where $K_1 = \sigma \rho_0 L_0 / m_p$ and $K_2 = \nu_0 L_0 / u_0$. Since we neglect the turbulent electric field $\mathbf{e}_m$, the corresponding terms in Equations (22) and (23) are omitted. The details of transforming the momentum and energy equations to the form of Equation (20) are given in Appendix B. The last (10th) equation in the system (20) is for the conservation of the total energy

$$E = \frac{\rho u^2}{2} + \frac{P E_u}{\gamma - 1} + \frac{3 P_I E_u}{2} + \frac{B^2}{2 M_A^2} - \rho \frac{\rho Z^2}{2}.$$

The equation for total energy is added to Equation (20) as a conservative alternative for the thermal pressure equation.

2.5. Numerical Scheme

To integrate the system of Equations (20) in time, we use the semi-discrete spatially third-order central weighted essentially non-oscillatory (CWENO) numerical scheme of Kurganov & Levy (2000) in combination with the third-order strong stability-preserving (SSP) Runge–Kutta time discretization of Gottlieb et al. (2001). The CWENO scheme is based on the idea of adaptive stencils and uses the local smoothness indicators and piecewise polynomial reconstructions to achieve high-order accuracy while avoiding oscillatory behavior near discontinuities. Each of the polynomial reconstructions is assigned a weight depending on the local smoothness of the solution, and the result is constructed as a weighted combination of the candidate reconstructions. To maintain the $\nabla \cdot \mathbf{B} = 0$ constraint, the time-dependent relaxation code employs the eight-wave method of Powell (1994).

2.6. Initial and Boundary Conditions

Since the turbulence transport model described in Section 2.2 assumes that $u_r \gg V_A$, we place our inner boundary far enough from the Sun to ensure that this condition is satisfied. The computational region extends out to 100 AU. It should be noted that although the model presented accounts for the interaction with the neutral hydrogen atoms that freely penetrate the heliospheric bubble, it does not involve the interaction with the ionized interstellar medium, nor does the model include the formation of the heliospheric boundaries (termination shock, heliopause, bow shock). Consequently, it is a pure solar wind model even at the outer boundary of 100 AU, chosen arbitrarily to restrict the computational domain.

To start the time-relaxation process, we need to specify an initial state by assigning initial values of all dependent quantities over the computational region. The initial conditions adopted for the present study are similar to those used by Usmanov et al. (2011). At $t = 0$, the plasma density, velocity, magnetic field, and thermal pressure are taken from a fully three-dimensional version of the tilted-dipole solar wind model of Usmanov & Goldstein (2003; see also Usmanov et al. 2000). Usmanov & Goldstein (2003) use a two-region model that consists of a time-relaxation “coronal” solution in the region from the coronal base out to 20 $R_\odot$ ($R_\odot$ is the solar radius) and its “heliospheric” counterpart with a solution constructed by the “marching-along-radius” integration of steady-state equations outward from 20 $R_\odot$ to 100 AU. The model is a wave-driven
model that incorporates a flux of Alfvén waves that is described statistically by the WKB approximation.

The waves propagate outward through a solar wind characterized by a polytropic index \( \gamma = 1.08 \) (1.46) in the “coronal” (“heliospheric”) region. The waves contribute to the acceleration of the wind to produce fast solar wind. The inputs to the model are the plasma and magnetic field parameters at the corona base: the driving amplitude of Alfvén waves (35 km s\(^{-1}\)), the initial (spherically uniform) density (0.4 × 10\(^6\) particle cm\(^{-3}\)) and temperature (1.8 × 10\(^6\) K), the intensity of the source solar magnetic dipole (16 G on the Sun’s pole), and its tilt with respect to the solar rotation axis. The source dipole means that the magnetic field is a dipole in the “coronal” region before the relaxation in this region begins. This initial configuration evolves as the relaxation proceeds. The solutions are constructed in the rotating solar equatorial coordinates, and the resulting solution is steady state in the rotating frame.

The initial distributions of density, velocity, thermal pressure, and magnetic field for this model are taken directly from Usmanov & Goldstein (2003). The initial distribution of the turbulent energy \( Z^2 \) is calculated from the WKB Alfvén wave energy density (Usmanov & Goldstein 2003). The initial conditions for \( \lambda \) and \( \sigma_c \) are assigned to be similar to those used by Breech et al. (2008) at 0.3 AU for all radial distances. (See Figure 1 in Usmanov et al. 2011 for the latitudinal boundary profiles of the plasma, magnetic field, and turbulence parameters at 0.3 AU.)

The density and temperature of pickup protons in the initial state are scaled as \( r \) from the values used by Whang (1998) at 1 AU (1.5 × 10\(^4\) particles cm\(^{-3}\) and 6.35 × 10\(^6\) K, respectively). In the course of the relaxation process, the initial state values assigned at the inner boundary are kept fixed. The outer boundary conditions at 100 AU were chosen to be of open type and approximated by a first-order (linear) extrapolation. Other model parameters used in this study are the same as those used by Breech et al. (2008) and Usmanov et al. (2011): \( \gamma = 5/3, \sigma_D = -1/3, \alpha = 2\beta = 0.8, f_D = 0.25, N_{1H0} = 0.1 \) cm\(^{-3}\), and \( L_{cav} = 8 \) AU.

The alignment of the dipole and the solar rotation axis implies that the dipole axis is perpendicular to the solar equatorial plane (“perpendicular dipole”). The solution therefore does not contain fast–slow stream interactions and is azimuthally and north–south symmetric. In that situation, the computational region was covered by the grid with 300 logarithmically spaced grid points along the radius, the latitudinal resolution of 1\(^\circ\), and only 18 points along longitude (the solution in the axisymmetric case is independent of longitude). For a tilted dipole, which is truly three-dimensional, there will be fast–slow stream interactions and CIRs will form. For that situation, the radial range was split into three sub-regions: 0.3–20, 20–60, and 60–100 AU, with 600 equidistant grid points along radius in each sub-region. The angular resolution in both latitude and longitude was then 1\(^\circ\):5.

2.7. Composite Grid

It is well known that solving the hydrodynamic or MHD equations in spherical coordinates, which is a natural choice in astrophysical and geophysical applications, is seriously complicated by the geometrical singularity at the poles of the form \( \sin^{-1} \) \( \theta \) (e.g., Roache 1972). In addition to the singularity, an even more serious problem is the convergence of meridians toward the poles with the linear spacing along \( \phi \) decreasing as \( \sin \theta \). As a result, higher grid resolution is applied to polar regions that are usually regular areas of the flow field. In time-dependent simulations, the small size of the near-pole grid cells can reduce significantly the global time step requirement imposed by the Courant–Friedrichs–Lewy condition.

If the areas around the poles are not excluded from a computation, a typical approach to circumvent the problem of singularity is to derive a set of reduced equations valid at \( \theta = 0, \pi \) using L’Hôpital’s rule (e.g., Griffin et al. 1979; Linker et al. 1991; Jablonowsky et al. 2006). The polar axis is then treated as a boundary, and the reduced equations are used as boundary conditions. Although this method solves the singularity problem, the problem of meridian convergence persists. A popular method that addresses both problems is the “cubed-sphere” grid consisting of six geometrically identical segments of quasi-spherical coordinates (Sadourny 1972; Ronchi et al. 1996). This mathematically elegant method requires rewriting the differential operators in non-orthogonal coordinates and involves also an inter-grid interpolation when applying boundary conditions for each segment. Its high programming overhead and the difficulties of parallelization are similar to those arising when unstructured grids (typically composed of triangles or tetrahedrons) are used.
Because a single coordinate system on a sphere always contains a singular point (Thorpe 1979), a natural approach is to introduce a composite grid composed of two or more fragments of spherical coordinates in which neither fragment contains polar singularities (Eby & Holloway 1994; Usmanov 1995, 1996; Kageyama & Sato 2004). The idea of rotating the spherical coordinate system by 90° to avoid the polar singularity and the convergence of meridians was first suggested by Semtner (1976). Eby & Holloway (1994) introduced a pair of spherical grids rotated relative to each other by 90°. Usmanov (1995, 1996) suggested using a composite grid consisting of three overlapping spherical grids. The first one is a spherical grid with a limited extension in latitude, and the other two are “patches” covering the polar regions in both hemispheres. The polar grids are fragments of a spherical coordinate system with its polar axis rotated by 90° and located in the equatorial plane of the first grid.

Figure 1(a) illustrates this approach, which we use in this paper. The figure shows the three overlapping grids (the main and polar grids are drawn with different radii for clearer presentation). Figure 1(b) shows the composite grid as it appears at each radial level from a point of view located above the north pole. The north and south grids are matched to optimize assigning boundary conditions at their latitudinal and longitudinal edges: their extreme points are fitted in between the inner points of the main grid that extended in latitude from −67.5° to +67.5° in our simulations. On the other hand, the extreme latitudinal points of the main grid are positioned among the inner points of the polar grids. Thus, after advancing the solution in the inner points of all the grids, the boundary conditions for each grid are assigned using the values interpolated from inner points of the overlapping grid(s).

We apply a simple bicubic interpolation between the main and polar grids at the overlapping grid interfaces. The procedure is non-conservative and involves interpolation errors. Although a conservative interpolation can be constructed (Chesshire & Henshaw 1994), the simple non-conservative interpolation has worked well in our simulations, even for strong shocks and discontinuities (see Henshaw 2010). The governing MHD equations have the same form for all the grids except for the terms involving solar rotation.

2.8. The Energy Equations

In computational hydro- and magnetohydrodynamics, it is often taken for granted that if shocks are present in the computational domain, the conservation form of governing equations should be used. The rationale for this is that the Rankine–Hugoniot relations are then embedded in the formulation and the conservation (flux) variables are continuous through shocks (e.g., Anderson 2009). Numerical experiments confirm that the conservation form leads to more accurate results for shock speed and intensity in some cases (e.g., Cocchi et al. 1998; Clarke 2010). There is, however, a specific difficulty that renders the conservation form of the energy equation inapplicable in many important cases.

The conservation form of the energy equation implies that the total energy of a flow element is preserved. When the thermal energy is not significantly (i.e., many orders of magnitude) smaller than other energy terms, the conservation form is preferable. However, if, e.g., magnetic or kinetic energy dominates thermal energy (the typical situation near the coronal base and beyond ∼20 R⊙, respectively), it is often impossible to calculate the thermal energy accurately because the calculation involves subtraction of large and nearly equal quantities (the magnetic or kinetic energy from the total energy). The problem is often discussed in the context of “positivity” of the thermal pressure that can at times become negative as a result of the subtraction. Some algorithms are aimed at preserving the “positivity” (e.g., Einfeldt et al. 1991; Hu & Khoo 2003). However, if a correct solution for thermal pressure is important, “positivity” might not be sufficient since it does not guarantee accuracy. At the same time, using the total energy equation is not problematic if thermal pressure remains positive and the focus is on the flow characteristics other than pressure or temperature (velocity, density, magnetic field), because the small pressure has little effect on the flow structure.

One approach out of this dilemma is the “dual energy” formulation that is based on switching to the conservation equation only in the vicinity of shocks (e.g., Ryu et al. 1993; Bryan et al. 1995; Balsara & Spicer 1999; O’Shea et al. 2005; Li et al. 2008). In addition to the computational overhead, this method is sensitive to the “switching conditions.” An alternative approach is just to use the energy equation in a non-conservation form.

In our study, we experimented with the conservation and non-conservation forms of the energy equation both separately and in the “dual energy” formulations of Ryu et al. (1993) and Cargill et al. (2000). We were unable to obtain a stable solution with the Ryu et al. switch conditions, while the Cargill et al. approach produced incorrect plasma temperatures in regions of large velocity shear. As a result, for the solutions described below, we have used the energy equation in a non-conservation form assuming that the possible inaccuracies in the vicinity of shocks are small. Note that the mass, momentum, and induction equations are fully conservative and that the equations for turbulent energy and cross-helicity cannot be cast in a conservation form.

3. SIMULATION RESULTS

3.1. Axially Symmetric Case

Near solar minimum, the dominant component of the coronal magnetic field is a dipole nearly aligned with the solar rotation axis (e.g., Hoeksema & Scherrer 1986; Zhao & Hoeksema 1996; Sanderson et al. 2003) and the heliosphere is azimuthally and north–south symmetric to a reasonable degree of approximation. The fast (slow) solar wind is confined to high (low) latitudes, and the heliospheric current sheet is aligned with the helioequatorial plane. In this (idealized) axisymmetric configuration, the solar wind structure is independent of longitude, there is no interaction between fast and slow wind streams, and the CIRs do not form.

Figure 2 shows radial variations of the solar wind and turbulence parameters near 30° north latitude from 0.3 to 100 AU for the source solar dipole aligned with the solar rotation axis. The deceleration effect is shown in Figure 2(a) by superposing the model profiles computed with effects of pickup protons (solid lines) and without the effects of pickup protons (dashed lines). At all distances, the pickup proton density Np is considerably smaller than the solar wind proton density Ns (Figure 2(c)). At 1 AU, the pickup proton pressure Pp is much smaller than the combined proton–electron pressure P and the magnetic pressure PM = B^2/8π (Figure 2(b)). However, Pp exceeds P and Pp outside ~5 AU because of a very high pickup proton temperature Tp (Figure 2(d)). The solar proton temperature Ts decreases monotonically with heliocentric distance, but the mean plasma temperature T = (N_s T_s + N_e T_e + N_p T_p) / (N_s + N_e + N_p)
increases. The small circles in Figures 2(b)–(d) are the values deduced from Voyager 2 observations by Burlaga et al. (1994, 1996). It should be noted that the profiles of $N_S$ in Figure 2(c) computed with and without pickup protons are virtually identical as the compression effect due to the deceleration is balanced by the loss of solar wind protons in the charge exchange process (Usmanov & Goldstein 2006). Note also that the deceleration effect of pickup protons due to mass/momentum loading is partially offset by the acceleration of the solar wind caused by the outward pressure gradient of pickup protons (Fahr & Fichtner 1995; Lee 1999; Fahr & Scherer 2004). We quantitatively estimated the acceleration effect by calculating the ratio of the acceleration and deceleration forces in the radial momentum (Equation (B3)), $(\partial P_t/\partial r)/(q_{ex} + q_{ph})m_p u_r$. We found this ratio to be less than 0.3% throughout the computational region. Although pickup protons dominate the thermal pressure in the distant heliosphere, the radial profile of $P_t$ is relatively flat (Figure 2(b)) and the internal energy of the pickup protons is small in comparison with the kinetic energy of the solar wind flow ($3P_t/\rho u^2 < 0.15$). As a result, the acceleration effect of the pickup proton pressure gradient is insignificant.

The magnetic field in the outer heliosphere is primarily azimuthal so that the deceleration causes a compression of the field intensity relative to the one that would be observed in the absence of the deceleration. Zank (1999) estimated that the increase in the azimuthal field compared to that expected from a constant velocity solar wind is of order of 10%. Figure 2(e) illustrates this by comparing the profiles of $B$ for the cases with and without pickup protons. The compression effect at this
Figure 3. Contour plots of the computed parameters in the meridional plane from 0.3 to 100 AU in the axisymmetric case: (a) the radial velocity $u_r$, (b) the number density $N_S$ of solar wind protons, (c) their temperature $T_S$, (d) the turbulent energy $Z^2$, (e) the cross-helicity $\sigma_c$, (f) the correlation length scale $\lambda$, (g) the number density $N_I$, and (h) the temperature $T_I$ of pickup protons. The white line in the $T_S$ plot depicts the projection of the Voyager 2 trajectory on the meridional plane.

particular latitude is up to $\sim$20% depending on the heliocentric distance. The turbulence parameters shown in Figures 2(f)–(h) are also significantly different in the presence of pickup proton effects. In particular, $Z^2$ decreases very slowly and $\sigma_c$, in contrast, relatively quickly beyond $\sim$10 AU.

Figure 3 shows contour plots of the mean-field and turbulent quantities in the meridional plane from 0.3 to 100 AU. The results are generally similar to those shown in Figure 3 in Usmanov et al. (2011). However, the radial velocity plot (Figure 3(a)) now shows the deceleration of the solar wind in the outer heliosphere due to the interaction with the interstellar hydrogen. In addition, the present model provides spatial distributions of the pickup proton density and temperature (Figures 3(g) and (h)). The distributions are similar to the ones for solar wind protons: the pickup proton density is higher and temperature is lower in the slow wind and vice versa in the fast wind. The white line in the $T_S$ plot represents the projection of the trajectory of Voyager 2 on the meridional plane to emphasize the variations the spacecraft would be expected to observe flying through the simulated structure.

Figure 4 illustrates the effect of pickup protons on the meridional profiles of solar wind parameters at the heliocentric distance of 80 AU. Again, the effects of deceleration, heating, and magnetic field compression are clearly seen in Figures 4(a), (d), and (e), respectively. The pickup protons have only a slight effect on the solar wind proton density as can be seen in Figure 4(c). Figure 4(e) shows the effect of the deceleration on the magnetic field in the meridional plane. The field magnitude is up to $\sim$30% higher than in the absence of pickup protons. While the pickup proton density and temperature depend on latitude (strongly in the slow wind near the equator and only slightly in the fast wind at higher latitudes), the thermal pressure of pickup protons is virtually uniform and almost two orders larger than the combined proton–electron pressure and the magnetic pressure.

3.2. Solar Cycle Variations

The evolution of the dipole component in the solar activity cycle with its reversals from one solar minimum to another suggests an idea to describe a solar cycle as a rotation of the dipole by 180° (Antonucci 1974; see also Balogh et al. 2008). Solar magnetic field observations generally support this approximation except for periods near solar maximum, when
a quadrupole term dominates the global solar magnetic field structure. In this section, we describe simulations for a number of tilt angles of the source dipole on the Sun.

Figure 5 shows contour plots in the meridional plane of the same variables as in Figure 3, with the addition of the meridional velocity $u_\theta$ and the magnetic field intensity $B$. The source dipole on the Sun is now tilted by 30$^\circ$. There are two sets of plots in this figure: the upper 10 plots illustrate the simulation results for the region from 0.3 to 20 AU, and the lower 10 for the region from 20 to 100 AU. Clearly, the flow structure is overwhelmingly more complex than that in the case without tilt. It is not axially or north–south symmetric anymore, and the solar rotation causes fast and slow streams to interact and form CIRs that develop into corotating shocks. The CIRs have the characteristic pattern of compressions that result from fast solar wind overcoming slow solar wind and rarefactions that appear when slow wind trails fast wind (see Hundhausen 1972, p. 135; Pizzo 1978). This phenomenon is seen clearly in the upper 0.3–20 AU plots. While the heliospheric neutral sheet (shown by white lines) oscillates within the angle of ±30$^\circ$, the flow characteristics are disturbed by interstream interaction within a considerably wider range of latitudes. This is a result of the compressions and rarefactions that form around the neutral sheet and extend to higher latitudes. Another factor is the meridional flows that tend to relax the compressions and fill in the rarefactions (compare Figure 5(b) with Figure 5(c)) and thus cause the interacting structures to reach even higher latitudes. Note that the corresponding azimuthal flows (not shown in Figure 5) redistribute the mass along longitude. The

latitudinally uniform fast wind is now confined to latitudes above ~45$^\circ$, while the slower wind band is structured by the CIRs. The deceleration of the fast wind is clearly seen in Figure 5(k).

Beyond ~20 $R_\odot$, the kinetic energy of the solar wind flow dominates all other forms of energy. As a result, the flow structure is almost exclusively determined by the distribution of velocity and density, while other flow quantities have little influence and serve more as tracers that follow closely the structure defined by the velocity and density. From the figure, it can be seen that the variations of the solar proton temperature $T_S$, the magnetic field intensity $B$, the turbulent energy $Z^2$, the density $N_I$, and the temperature $T_I$ of pickup protons are all structured similarly to the solar proton density $N_S$. It is interesting to note that the distribution of the cross-helicity $\sigma_c$ remains in general similar to the one in the case of non-tilted dipole (cf. Figures 5(q) and 3(e)) as the cross-helicity differs noticeably from zero only around the polar axis.

Figure 6 presents the simulation results for the source dipole on the Sun tilted by 60$^\circ$. The fast wind is confined now to a narrow region around the polar axis. The slower wind structured by CIRs occupies the major portion of the heliosphere, and the meridional flows generated by the CIRs approach the pole very closely. A truly three-dimensional effect of pickup protons in the distant heliosphere is that, as a result of the deceleration of the solar wind and the additional pressure exerted by the pickup protons, they act to weaken the CIRs. Figure 7 compares the radial profiles of the solar proton density $N_S$ scaled as $r^2$ and the magnetic field magnitude $B$ scaled as $r$ for the cases with and

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**Figure 4.** Meridional profiles at $r = 80$ AU of (a) the radial velocity $u_r$, (b) the pressure $P$, the pickup proton pressure $P_I$, and the magnetic pressure $P_M$, (c) the solar proton density $N_S$ and the pickup proton density $N_I$, (d) the solar proton temperature $T_S$ and the pickup proton temperature $T_I$, (e) the magnetic field magnitude $B$, and (f) the turbulent energy $Z^2$. The dashed lines show the results with all the effects of pickup protons turned off. The source dipole on the Sun is aligned with the solar rotation axis.
Figure 5. Contour plots in the meridional plane from 0.3 to 20 AU (a–j) and from 20 to 100 AU (k–t) of the mean-flow and turbulence parameters for the source dipole on the Sun tilted by 30° with respect to the solar rotation axis. The white wavy lines depict the heliospheric neutral sheet ($B_r = 0$). The white line in the $T_S$ plot is the projection of the Voyager 2 trajectory on the meridional plane.
Figure 6. Same as in Figure 5 for the source dipole on the Sun tilted by 60°.
tilts of 0° spacecraft. The model profiles correspond to the solar dipole in comparison with the proton temperature observed by the trajectory of Voyager 2. A 27 day running average was applied to both the model and Voyager 2 data to filter out the variations associated with the solar rotation. The upper plots display the model variations along the actual Voyager 2 trajectory (shown in the projection on a meridional plane in Figure 3(c)) and along the projection of the Voyager trajectory on the equatorial plane (as if the spacecraft would remain in the equatorial plane while moving outward). In general, the model profiles in Figure 8(a) computed for various tilts of the solar dipole are consistent with the range of temperatures observed by the spacecraft.

While the model variations for the tilts of 60° and 90° are almost identical, those for 0° and 10° are significantly different outside 30 AU. The interpretation of the difference is straightforward if we recall that the small tilts are representative of solar minimum. Ulysses observations have revealed that near solar minimum the heliospheric structure is strongly latitude-dependent with a slow wind around helioequator and a fast wind at higher latitudes (Phillips et al. 1995). The proton temperature of the fast wind measured by Ulysses was typically higher than the temperature of the slow wind (Goldstein et al. 1996) (in agreement with the well-known correlation between solar wind speed and proton temperature; Burlaga & Ogilvie 1970). This fact is consistent with the increase of proton temperature with latitude in our model (see Figure 3(c)). Thus, the profiles presented in Figure 8(a) as functions of radius are in fact mixed radial and latitudinal variations. The displacement of Voyager 2 to higher southern latitudes after 30 AU and its gradual immersion into faster wind are presumably responsible for the apparent inflection in the observed proton temperature at just about 30 AU and could contribute to the flattening of the temperature presented as a function of radius. On the other side, little difference between the model profiles for the tilts of 60° and 90° in Figures 8(a) and (b) indicates that they have little latitude dependence. The tilts of 60° and 90° are representative of periods around solar maxima when the slow

### Figure 7
Radial profiles of (a) the solar proton density (scaled as $r^2\dot{N}_p$) and (b) the magnetic field magnitude (scaled as $r B$) near the helioequator ($\theta = 90.75^\circ$) showing the effect of the reduced compression in the corotating interaction regions due to pickup protons. The source dipole on the Sun is tilted by $30^\circ$ without pickup protons. The pickup protons work to alleviate the CIRs, which is clearly seen in Figure 7.

#### 3.3. Comparison with Voyager 2

Figure 8 shows variations of the model temperature along the trajectory of Voyager 2 as a function of heliocentric distance in comparison with the proton temperature observed by the spacecraft. The model profiles correspond to the solar dipole tilts of 0°, 10°, 30°, 60°, and 90°. A 27 day running average was applied to both the model and Voyager 2 data to filter out the variations associated with the solar rotation. The upper plots display the model variations along the actual Voyager 2 trajectory (shown in the projection on a meridional plane in Figure 3(c)) and along the projection of the Voyager trajectory on the equatorial plane (as if the spacecraft would remain in the equatorial plane while moving outward). In general, the model profiles in Figure 8(a) computed for various tilts of the solar dipole are consistent with the range of temperatures observed by the spacecraft.

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solar wind occupies the entire heliosphere with little dependence of latitude (see Figure 6(n)).

The temperature profiles computed for the tilt of 10° without any turbulence effects (the turbulence transport equations are excluded from the governing system and the turbulent energy $Z^2$ is set everywhere to zero), without the effect of pickup protons (the pickup proton production rates $q_{ph} = q_{ex} = 0$), and without both the effects of turbulence and pickup protons are presented in Figure 8(c). Comparison of the solutions demonstrates that beyond $\sim 30$ AU the heating by pickup protons dominates the effect of the turbulent cascade (Williams et al. 1995) and that, in the absence of turbulence, the effect of pickup protons on the temperature distribution is relatively small.

4. SUMMARY AND FUTURE WORK

We have developed a fully three-dimensional solar wind model that accounts for transport of turbulence and treats pickup protons as a separate fluid. The model is based on a numerical solution of the coupled set of the mean-field Reynolds-averaged solar wind equations with pickup protons and the turbulence transport equations in the region from 0.3 to 100 AU. The pickup proton distribution is assumed to comove with the solar wind flow and is described by separate mass and energy equations. The governing equations account for the energy transfer from pickup protons to solar wind protons and for the plasma heating by turbulent dissipation.

We have shown that the structure of the heliosphere is strongly affected by the interstellar pickup protons. Although the pickup proton population is a relatively minor constituent of the interplanetary medium near Earth’s orbit, its role becomes increasingly important farther out in the heliosphere ($r > 5$–10 AU), where the pickup protons dominate the thermal pressure of the solar wind plasma.

There are several important effects of pickup protons that we simulated in this study: the deceleration of the solar wind as a result of the momentum transfer from the solar wind protons to pickup protons, the heating of solar wind plasma by the dissipation of turbulent energy, the compression of predominantly azimuthal magnetic field in the outer heliosphere as a result of the deceleration, and the weakening of the CIRs due to the deceleration and the thermal pressure of pickup protons. Comparing results from the simulation runs with and without pickup protons, we evaluated and compared these effects quantitatively.

Contrasting the profiles of the proton temperature computed along the Voyager 2 trajectory for a number of tilts of the source dipole on the Sun (representing various phases of solar cycle), we demonstrated that the range of the model profiles of solar proton temperatures is generally consistent with Voyager 2 observations. The latitudinal displacement of the spacecraft after the spacecraft encounter with Neptune in 1989 is an important factor in interpretation of its observations.

There are a number of limitations to the model in its present form that we are planning to address in future work. One deficiency is that the protons and electrons are treated as a single fluid and their thermal pressures are assumed to be equal. This implies that the turbulent heating is equally partitioned between the protons and electrons, as is the transport of heat due to the electron heat conduction. The natural extension to the present model will be to describe protons and electrons by separate energy equations (Breech et al. 2009). Another limitation is that the modeling of the Reynolds stresses uses the relatively simple approach based on an assumed symmetry of the fluctuations.

We are currently developing an eddy-viscosity approximation that can be incorporated into the model that will provide a more adequate description of the energy transfer between mean-field flow and the turbulence. An important restriction imposed by the present formulation of the turbulence transport model is that the solar wind model presented in this paper is limited to the super-Alfvénic solar wind regime. Our intention is to relax the assumption $u_r \gg V_A$ (see Zank et al. 2011) and replace the WKB Alfvén wave approach within 0.3 AU with a trans-Alfvénic turbulence transport model. Another improvement will be incorporating the direction of the inflow of the interstellar hydrogen from the interstellar medium.

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APPENDIX A

EQUATIONS FOR ELÁSSER VARIABLES IN THE ROTATING FRAME IN THE PRESENCE OF PICKUP PROTONS

Using Equations (8) and (12), the Reynolds-averaged continuity equation for the total mass density, $\rho = m_p (N_S + N_I)$, can be cast into the form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = q_{ph} m_p. \tag{A1}$$

In the case of locally incompressible fluctuations, $\rho' \equiv 0$, subtracting Equation (A1) from the equation $\frac{\partial \rho' }{\partial t} + \nabla \cdot (\rho' \mathbf{v}) = q_{ph} m_p$, which is a corollary of Equations (1) and (5), one gets the relation between the gradient of density and the divergence of fluctuating velocity: $(\nabla \cdot \mathbf{v})' = -\rho (\mathbf{v} \cdot \nabla)'. \tag{A2}$

Subtracting Equation (10) from Equation (3), one arrives at

$$\frac{\partial \mathbf{b}'}{\partial t} = (\mathbf{b}' \cdot \nabla)\mathbf{v} + (\nabla \cdot \mathbf{V}_A)\mathbf{v}' - (\nabla \cdot \mathbf{v})\mathbf{b}'$$

$$- (\mathbf{v} \cdot \nabla)\mathbf{v}_A - \frac{\mathbf{b}'}{2} (\mathbf{v} \cdot \nabla) - \frac{\mathbf{b}'}{2} \frac{q_{ph} m_p}{\rho}$$

$$+ (\mathbf{b} \cdot \nabla)\mathbf{v}' - (\mathbf{v} \cdot \nabla)\mathbf{b}' - \langle (\mathbf{b}' \cdot \nabla)\mathbf{v}' \rangle + \langle (\mathbf{v} \cdot \nabla)\mathbf{b}' \rangle$$

$$- \frac{1}{2} \mathbf{b}' (\nabla \cdot \mathbf{v}') + \frac{1}{2} \langle (\mathbf{b}' \cdot \nabla)\mathbf{v}' \rangle\rangle,$$
where $\tilde{P}_v$ is the fluctuation of the total of thermal and magnetic pressure $P_v = P' + P' + B_z^2 + 2 \langle B \cdot B' \rangle - \langle B'^2 \rangle / 8 \pi$. Using the vector identity $(A \cdot \nabla)(\Omega \times r) = \Omega \times A$, where $A$ is an arbitrary vector, it is straightforward to rewrite Equations (A2) and (A3) using the Elsässer variables, $z^\pm = v^\pm b^\pm$, in the form

$$\frac{\partial z^\pm}{\partial t} + [(v \mp \nabla \cdot z^\pm + z^\mp \cdot \nabla)(u \pm \nabla v)]$$

$$+ \frac{1}{2} \frac{z^\pm - z^\mp}{2 \rho} \left[ \nabla \cdot \left( \frac{u^\pm}{2 \rho} + \frac{q_{ph} m_p}{2 \rho} \right) + \frac{v \mp \nabla A}{2 \rho} \right] = \text{NL}^\pm + S^\pm,$$

and $S^\pm$ is added to represent any source terms.

**APPENDIX B**

THE MOMENTUM AND ENERGY EQUATIONS WITH PICKUP PROTONS IN THE ROTATING FRAME

The momentum equation with the deceleration term due to pickup protons (9) can be written for the transverse to $B$ and axisymmetric turbulence in the form

$$\frac{\partial (\rho v)}{\partial t} + \nabla \cdot \left( \rho vv - \frac{\eta}{4 \pi} BB + \tilde{P} I \right)$$

$$+ \frac{\partial}{\partial r} \left[ \frac{GM_c}{r^2} \hat{r} + 2 \Omega \times v + \Omega \times (\Omega \times r) \right]$$

$$= -m_p q_{ex} u + q_{ph} \Omega \times r,$$

(B1)

where $\tilde{P} = P + P' + (1 + \sigma_D) \rho Z^2 / 4 + B_z^2 / 8 \pi$ is the total pressure and $\eta = 1 + 2 \pi \sigma_D \rho Z^2 / B_z^2$. Using the relation $\nu = u - \Omega \times r$, where $u$ is the velocity in the inertial frame, Equation (B1) can be transformed as

$$\frac{\partial (\rho u)}{\partial t} + \nabla \cdot \left( \rho vu - \frac{\eta}{4 \pi} BB + \tilde{P} I \right)$$

$$= -q_{ex} m_p u - \rho \left( \frac{GM_c}{r^2} \hat{r} + \Omega \times u \right),$$

(B2)

A non-conservative form of Equation (B2) can be obtained using the continuity Equation (A1)

$$\rho \left[ \frac{\partial u}{\partial t} + (v \cdot \nabla)u \right] - \frac{1}{4 \pi} (\nabla \times B) \times B$$

$$+ \nabla \cdot \left[ P + P' + \frac{(1 - \sigma_D) \rho Z^2}{4} + \frac{\sigma_D \rho Z^2}{2} (1 - \hat{B} B) \right]$$

$$= -\left( q_{ex} + q_{ph} \right) m_p u - \rho \left( \frac{GM_c}{r^2} \hat{r} + \Omega \times u \right),$$

(B3)

Now with the use of Equations (A1), (B3), (10), (11), (13), and (15), we can derive a conservation equation for the total energy

$$E = \frac{\rho u^2}{2} + \frac{P}{\gamma - 1} + \frac{3 P_1}{2} + B_z^2 - \rho \frac{GM_c}{r} + \rho \frac{Z^2}{2},$$

(B4)

in the form

$$\frac{\partial E}{\partial t} = -\nabla \cdot \left[ \nu E + u \tilde{P} - \frac{\eta B(u \cdot B)}{4 \pi} + q_{ph} I \right] - q_{ex} m_p \frac{GM_c}{r}$$

$$- \frac{\alpha f^*(\sigma_r) \rho Z^3}{4}$$

$$- \frac{\eta B}{4 \pi} \times (\nabla \times \epsilon_m),$$

(B5)

where $\rho_S = m_p N_S$. The equation for thermal pressure (11) with the Hollweg heat flux $q_{ex} = (3/4) a \nu P$ and the turbulent heating $Q_1(r) = (\gamma - 1) \alpha f^*(\sigma_r) \rho Z^3 / 2 \lambda$ can be rewritten as

$$\frac{\partial P}{\partial t} + \frac{\gamma - 1}{\gamma - 1} \nu (v \cdot \nabla)P + \gamma_1 P \nu \cdot v = \frac{(\gamma - 1) \alpha f^*(\sigma_r) \rho Z^3}{2 \lambda}$$

$$- \frac{\gamma P}{N_S} q_{ex},$$

(B6)

where

$$\gamma_1 \equiv \gamma + (3a/4)(\gamma - 1) / (1 + (3a/4)(\gamma - 1))$$

is an effective polytropic index that depends on Hollweg’s constant $a$ and differs from $\gamma$ only if $a \neq 0$. For steady-state flows and in the absence of pickup proton effects and additional heat sources/sinks, Equation (B6) shrinks to $(v \cdot \nabla)P + \gamma_1 P \nu \cdot v = 0$, i.e., the Hollweg conduction term is mathematically identical in this case to a non-adiabatic polytropic index (Jacques 1978; Meyer-Vernet 2007). For instance, if $\gamma = 5/3$ and $a = 1.05$ (Cramer et al. 2009), then $\gamma_1 = 1.44$. Using the identity $(v \cdot \nabla)P + \gamma_1 P \nu \cdot v = \gamma_1 P^{1-(1/\gamma_1)} \nu \cdot (P^{1/\gamma_1} v)$, we can transform Equation (B6) into the following conservative form (see Ryu et al. 1993):

$$\frac{\partial P^{1/\gamma_1}}{\partial t} + \gamma_1 (v \cdot \nabla) (P^{1/\gamma_1} v) = \frac{1}{\gamma_1}$$

$$\times \left[ \gamma_1 q_{ex} / N_S - (\gamma - 1) \alpha f^*(\sigma_r) \rho Z^3 / 2 \lambda P \right].$$

(B7)

A similar transformation of Equation (13) gives

$$\frac{\partial P^{3/5}}{\partial t} + (v \cdot \nabla) P^{3/5} = \frac{1}{5} P^{2/5} \left[ (q_{ex} + q_{ph}) m_p \right.$$

$$\times \left. \left[ u^2 + (1 + \sigma_D) Z^2 / 2 \right] - \rho \tilde{E} \right],$$

(B8)

REFERENCES


