ISSN 0016-7932, Geomagnetism and Aeronomy, 2015, Vol. 55, No. 7, pp. 902–906. © Pleiades Publishing, Ltd., 2015.

The Gnevyshev-Ohl Rule and Its Violations[1]

N. V. Zolotova and D. I. Ponyavin

Saint-Petersburg State University, Saint-Petersburg

e-mail: ned@geo.phys.spbu.ru, ponyavin@geo.phys.spbu.ru

Received February 8, 2015; in final form, February 13, 2015

Abstract—The 22-year cycle in the solar activity known as the even-odd effect is compared with the odd-even order. We find that the grouping into the even-odd pairs is not preferable in comparison to that of odd-even ones. We argue that due to existed relations between any adjacent cycles, combination of solar cycles in pairs according to their numbers is not justified.

DOI: 10.1134/S0016793215070300

1. INTRODUCTION

It is well known that solar cycles are different according to their amplitude, length or intensity. The last one is the sum of sunspot numbers/groups over a cycle (from minimum to minimum). It is currently accepted that there is a 22-year modulation of the solar cycles in regard to the even-odd cycle pairing. Analyzing Wolf's sunspot numbers from 1842 to 1923 (Cycles 9–15, according to the Zürich numbering), Turner (1913, 1925) found that the odd Cycles 9, 11, and 13 are on average higher than the even Cycles 10, 12, and 14 correspondingly. Also, the spots of the odd cycles are on average 1° further from the equator in both hemispheres, and the sunspot area is larger than that of the even cycles. He concluded that a major sunspot period is a double period of 23 years. Thus, he proposed that cycles should be organized in pairs starting from the stronger odd cycle: 9–10, 11–12, 13–14, 15–....

Analyzing sunspot numbers for Cycles –4–17, Gnevyshev and Ohl (1948) found a close statistical relationship between the even-odd numbered cycles, while a linear correlation of the odd–even cycles was weak. Often this empirical regularity is considered as an equivalent of the even-odd effect, and it is called the Gnevyshev-Ohl (GO) rule (Hathaway, 2010). Currently, the even-odd effect is defined as follows: the sum of sunspot numbers over the even cycle is less than that of the following odd cycle.

In order to check the even-odd effect, various solar activity indices have been used (Nagovitsyn et al., 2009; Petrovay, 2010; Ogurtsov and Lindholm, 2011; Tlatov, 2013). To process data for the period before 1850, Usoskin et al. (2001) and Tlatov (2013) propose to use the group sunspot number (R_g) introduced by Hoyt and Schatten (1998) as more homogeneous series. Mursula et al. (2001) reported that before the Dalton minimum, the even-odd effect is valid in a phase-reversed form when an odd cycle is followed by a more intense even cycle. Having processed the daily average number of sunspot groups in a cycle, Tlatov (2013, 2015) suggested that the GO rule is reversed between Cycles 9 and 10 and between Cycles 21 and 22, and it also exhibits the long-term variation with a period of about 230 years (21 cycles) with the exception of Cycles 6 and 7 of the Dalton minimum. In accordance with Mursula et al. (2001), Tlatov (2013) reported that during the period 1745–1850 in the even-odd pairs the odd cycles had a lower daily average number of sunspot groups in a cycle than did the preceding even cycles.

To restore the GO rule, Usoskin et al. (2001, 2009) hypothesize that a low and short sunspot cycle was possibly lost at the beginning of the Dalton minimum. However, Zolotova and Ponyavin (2011) supposed that Cycle 4 should not be divided into two cycles, and this strong cycle was prolonged due to an impulse of activity in the northern hemisphere during the descending phase.

In this paper, we consider pairs of cycles in respect to the even-odd grouping versus the odd-even one. We call the even-odd pairing of the solar cycles as the Gnevyshev-Ohl order and the odd-even one, the Turner order.

2. ODD-EVEN AND EVEN-ODD ORDERS OF SOLAR CYCLES

Let us compare the even-odd and odd-even pairing of the solar cycles. Figure 1 demonstrates the yearly group sunspot numbers (R_g) from 1610 to 1995 supplemented with yearly sunspot numbers (R_i) from 1996 to 2012. Notice that Cycle 0 has a gap in observations from 1744 to 1747. However, the shape of this cycle is found to be the most similar to that of Cycle 1. Thus, the first four years of Cycle 0 are filled with the yearly group sunspot numbers starting the onset of Cycle 1. Based on resemblance of Cycles 14 and 24 (Solanki

[1] The article is published in the original.

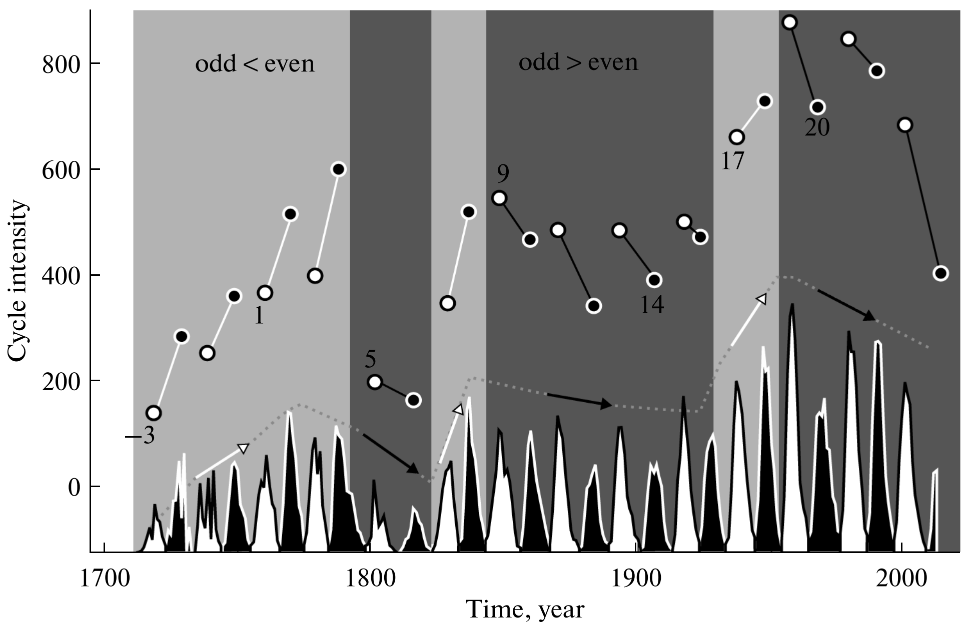

Fig. 1. Solar cycles in the course of the Gleissberg cycle. Dotted line schematically shows the secular modulation. White arrows indicate the ascending phase of the Gleissberg cycle, black ones, the descending phase. Odd cycles are painted white, even ones, black. Cycle intensities are shown by circles. Each pair of odd-even cycles is joined by a white segment in case when odd cycles are weaker than even ones, and black color denotes the opposite situation. Numbers refer to the Zürich numbering. The light gray zones join pairs, when the intensity of the preceding cycle is less than that of the following cycle, the dark gray zones, preceding > following.

and Krivova, 2011), the intensities of these cycles are suggested to be similar as well. Odd cycles colored white, even ones, black. Dotted line schematically shows the secular modulation. White arrows indicate the ascending phase of the Gleissberg cycle (Gleissberg 1939, 1945), while black ones, the descending phase. We do not consider Cycle –4, since it is strongly asymmetric relatively to the equator (Ribes and Nesme-Ribes, 1993). In Fig. 1, above R_g, the intensities of the odd and even cycles are shown in pairs. For Cycles –3 to Cycle 4, the intensity of an odd (white) cycle is lower than that of the following even (black) cycle (the ascending phase of the Gleissberg cycle). The pair of Cycles 5–6 belongs to the declining phase of the secular cycle. The intensity of Cycle 5 is higher than that of the next cycle, and so on for the ascending and declining phases of the Gleissberg cycle. Similarly to Tlatov (2013), we define the reversal into a cycle pairing when the intensity of the first cycle in a pair became lower or higher than that of the second cycle (transitions between light and dark gray bars).

Figure 2 illustrates the same intensities of the yearly R_g cycles, when the last ones are combined into the even-odd pairs. For Cycles 6–21 the intensity of an even cycle is lower than that of the following odd cycle, which is in accordance with the Gnevyshev-Ohl rule (1948).

3. DISCUSSION

Comparing Figs. 1 and 2, one can see that the asymmetry of intensities in the odd-even pairs tend to reverse near the extrema of the secular cycle (transitions between light and dark gray bars). However, the reversals in the even-odd pairs do not reflect the secular modulation. Figure 3 shows the intensities of cycles for the yearly R_g (black circles) and R_i (gray circles). Here, time phases, when the following cycle is weaker than the preceding one, are marked by dark gray bars, and vice versa (the following cycle is stronger than the preceding one), by light gray bars. Beginning from the Cycle 3, R_g and R_i shows synchronous growth or decline of the intensities of adjacent cycles. Behavior of the nearest cycles can be divided into three types: alternation of weaker and stronger cycles (e.g. Cycles 9–16), consecutive growth (Cycles 6–9), consecutive decline (Cycles 4–6). The alternation of weaker and stronger cycles corresponds to 22-year period, consecutive growth and decline, the ascending and descending phase of the secular cycle. Each transition from one type of behavior to another causes a reversal in the even-odd (Gnevyshev and Ohl, 1948) and odd-even (Turner, 1913, 1925) orders. Thus, the reversals of a cycle pair take place, when asymmetry of the intensities of the nearest cycles changes.

Any attempts to reveal statistical regularity in cycle pairing and a physical interpretation, which would be responsible for this phenomenon, inevitably leads to the violations: pair of Cycles –2 and –1 (Nagovitsyn

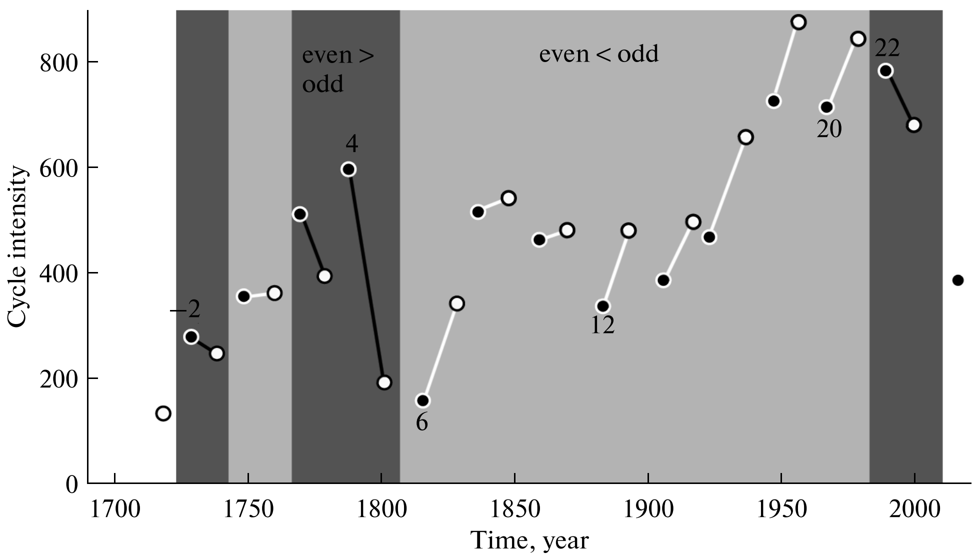

Fig. 2. The same as in Fig. 1, but for the even-odd pairing of the solar cycles.

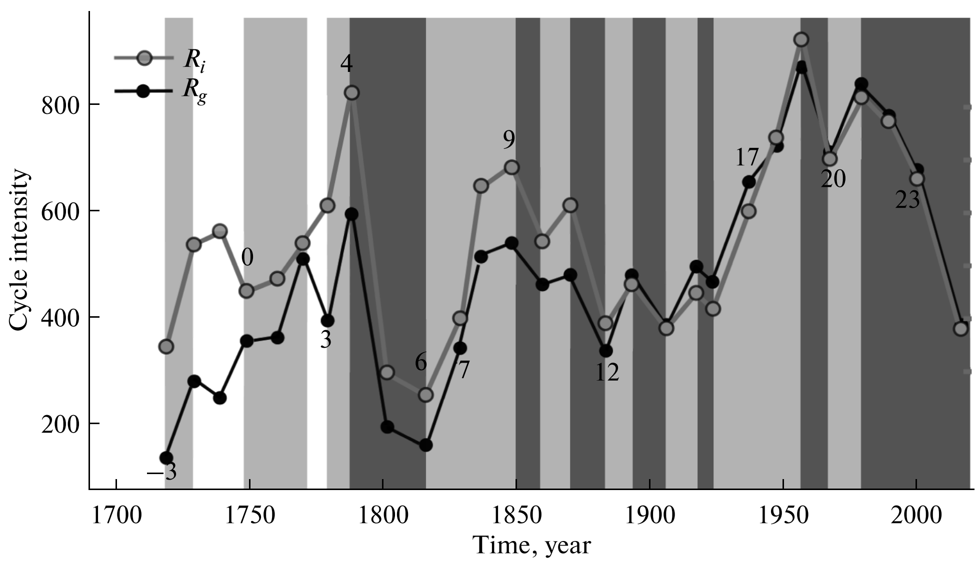

Fig. 3. The intensities of cycles for the yearly R_g (black circles) and R_i (gray circles). Numbers refer to the Zürich numbering. The light gray zones join pairs, when the following cycle is stronger than the preceding one, and dark gray zones, vice versa (the following cycle is weaker than the preceding).

et al., 2009), pair 4–5 (Gnevyshev and Ohl, 1945), pair 6–7 (Tlatov, 2013), pair 8—9 (Hathaway, 2010), and pair 22–23 (Hathaway, 2010; Tlatov, 2013).

It is obviously evident that lack of statistics crucially influences the results. Therefore, verification of the Gnevyshev-Ohl rule strongly depends on the time-series length. Due to unsufficient statistics the correlation coefficient significantly depends on each event. For Cycles –4–17, the linear correlation between the set of even cycles preceding that of odd cycles doubles (Gnevyshev and Ohl, 1945), when pair 4–5 is excluded from analysis. Similarly, the linear correlation between the set of odd cycles preceding even cycles strongly depended on the time-series length. Therefore, for the time-series from 1700 to 1944, the linear correlation of the odd-even cycles is weak, however, for the time-series up to Cycle 23, the correlation of the odd-even pairs is found to be higher than that for the even-odd ones (Tlatov, 2013; Zolotova and Ponyavin, 2014a).

All these findings suggest to us that a preference can not be given neither the even-odd order nor the odd-even one. To emphasize this conclusion, we made a

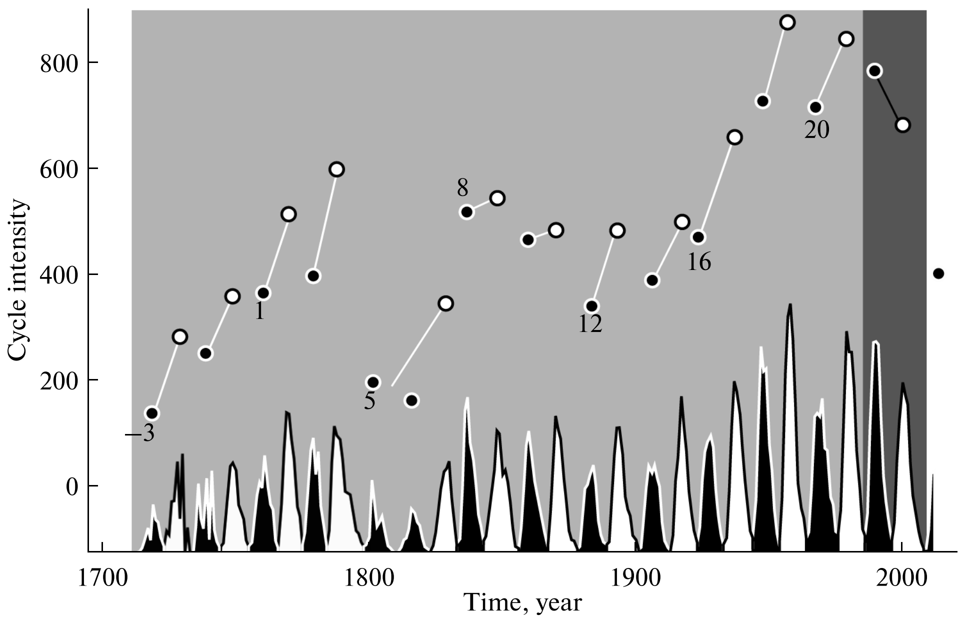

Fig. 4. The same as in Figure 1, but without the polar field reversal at the maxima of Cycle 5. Intensities of sunspot cycles in pair are colored according to Hale's law.

thought experiment to mix the even-odd and odd-even orders.

If variations of the sunspot activity are associated with those of the polar field reversals (Zolotova and Ponyavin, 2012; Svalgaard and Kamide, 2013), then the polar field budget should reflect the cycle intensity. In other words, strong cycles generate the strong polar field, while weaker cycles, weaker polar field (Zolotova and Ponyavin, 2014b). Let us speculate that an activity budget of the weak Cycle 5 is not enough to reverse a strong polar field from the previous cycle. Callebaut et al. (2007) suggested that the yearly mean sunspot number has to exceed 40 in order to make polar reversals. We should also notice that the annual sunspot number For Cycles 5 and 6 is about 45. The hypothetical breaking of the polar field reversal at the maximum of Cycle 5 implies that successive Cycles 5 and 6 have identical "polarity patterns" corresponding to Hale's law violations. Figure 4 demonstrates this hypothesis in a context of solar cycles throughout the 300-year interval of sunspot observations. Remind that a similar "restoration" of the Gnevyshev-Ohl rule was proposed by means of a division of Cycle 4 into two short cycles (Usoskin et al., 2001, 2009; Nagovitsyn et al., 2009). On the bottom of Fig. 4, we color cycles not according to the even-odd effect, but in accordance with the polarity sign in respect to Hale's law. During the century before the Dalton minimum, the white cycles are even-numbered and dominate the preceding black (odd) ones, then, after the Dalton minimum, in each pair of cycles an odd cycle dominates the preceding even one, but in the pair of Cycles 22–23 this regularity is violated (reversed). Thus, we made a toy situation, where the odd-even pairing is transformed into the even-odd ones. All our findings illustrate that cycles can be combined by different ways, and therefore, suggest to us that the cycle pairs in referring to the cycle number does not have a sense.

4. CONCLUSIONS

In this paper, we consider an asymmetry of intensities in pair of sunspot cycles. We define the even-odd pairing of solar cycles as the Gnevyshev-Ohl (1948) order and the odd-even ones, the Turner (1913, 1925) order. Analysing the reversals of a cycle pair (when the first cycle in a pair became weaker or stronger than the second one), we conclude that the reversals take place, when the type of behavior of the nearest cycles changes. We mark out three types of behavior: alternation of weaker and stronger cycles (22-year modulation), consecutive growth or consecutive decline (secular modulation). Both odd-even and even-odd pairs demonstrate similar correlations, so the preference can not be given to any of them (neither the even-odd nor the odd-even order). These findings suggest to us that there is relation between any adjacent cycles, but combining solar cycles in pairs according to their numbers lacks a physical basis.

We use: yearly group sunspot numbers proposed by the NOAA's National Geophysical Data Center (NGDC) (http://www.ngdc.noaa.gov/stp/spaceweather.html); yearly sunspot numbers, proposed by the Solar Influences Data Analysis Center (SIDC) (http://sidc.oma.be/index.php).

The reported study was partially supported by RFBR, research project no. 15-02-06959 A.

REFERENCES

Callebaut, D.K., Makarov, V.I., and Tlatov, A.G., Monopolar structure of the Sun in between polar reversals and in Maunder Minimum, *Adv. Space Res.*, 2007, vol. 40, pp. 1917–1920.

Gleissberg, W., A long-periodic fluctuation of the sun-spot numbers, *Observatory*, 1939, vol. 62, pp. 158–159.

Gleissberg, W., Evidence for a long solar cycle, *Observatory*, 1945, vol. 66, pp. 123–125.

Gnevyshev, M.N. and Ohl, A.I., On 22-year cycle of the solar activity, *Astronomicheskii Zhurnal*, 1948, vol. 25, pp. 18–20.

Hathaway, D.H., The solar cycle, *Living Rev. Solar Phys.*, 2010, vol. 7, no. 1, p. 65.

Hoyt, D.V. and Schatten, K.H., Group sunspot numbers: a new solar activity reconstruction, *Sol. Phys.*, 1998, vol. 179, pp. 189–219.

Mursula, K., Usoskin, I.G., and Kovaltsov, G.A., Persistent 22-year cycle in sunspot activity: evidence for a relic solar magnetic field, *Sol. Phys.*, 2001, vol. 198, pp. 51–56.

Nagovitsyn, Y.A., Nagovitsyna, E.Y., and Makarova, V.V., The Gnevyshev-Ohl rule for physical parameters of the solar magnetic field: The 400-year interval, *Astron. Lett.*, 2009, vol. 35, no. 8, pp. 564–571.

Ogurtsov, M. and Lindholm, M., Statistical effects in the solar activity cycles during AD 1823–1996, *ISRN Astron. Astrophys.*, 2011, vol. 2011, Article ID 640817.

Petrovay, K., Solar cycle prediction, *Living Rev. Solar Phys.*, 2010, vol. 7, no. 6, p. 59.

Ribes, J.V. and Nesme-Ribes, E., The solar sunspot cycle in the Maunder minimum AD1645 to AD1715, *Astron. Astrophys.*, 1993, vol. 276, pp. 549–563.

Solanki, S.K. and Krivova, N.A., Analysing solar cycles, *Science,* 2011, vol. 334, pp. 916–917.

Svalgaard, L. and Kamide, Y., Asymmetric solar polar field reversals, *Astrophys. J.*, 2013, vol. 763, article id. 23 (pp. 6).

Tlatov, A.G., Reversals of Gnevyshev-Ohl rule, *Astrophys. J.*, 2013, vol. 772, no. 2, p. L30.

Tlatov, A.G., The change of the solar cyclicity mode, *Adv. Space Res.*, 2015, vol. 55, pp. 851–856.

Turner, H.H., On a simple method of detecting discontinuities in a series of recorded observations, with an application to sun-spots, suggesting that they are caused by a meteor swarm, *MNRAS*, 1913, vol. 74, pp. 82–109.

Turner, H.H., Note on the alternation of the eleven-year solar cycle, *MNRAS,* 1925, vol. 85, pp. 467–471.

Usoskin, I.G., Mursula, K., Arlt, R., and Kovaltsov, G.A., A solar cycle lost in 1793–1800: early sunspot observations resolve the old mystery, *Astrophys. J.*, 2009, vol. 700, p. L154.

Usoskin, I.G., Mursula, K., and Kovaltsov, G.A., Was one sunspot cycle lost in late XVIII century?, *Astron. Astrophys.*, 2001, vol. 370, p. L31.

Zolotova, N.V. and Ponyavin, D.I., Enigma of the solar cycle 4 still not resolved, *Astrophys. J.*, 2011, vol. 736, article id. 115 (pp. 5).

Zolotova, N.V. and Ponyavin, D.I., Reconstruction of magnetic field surges to the poles from sunspot impulses, *Proceedings of the International Astronomical Union, IAU Symposium 286*, Mandrini, C.H. and Webb, D.F., Eds., Cambridge Univ. Press, 2012, pp. 88–92.

Zolotova, N.V. and Ponyavin, D.I., On the Gnevyshev-Ohl rule and its violations, *Proceedings of the "Solar and solar-terrestrial physics–2014"*, Stepanov, A.B. and Nagovitsyn, Y.A., Eds., Saint-Petersburg, 2014a, pp. 157–160. (in Russian).

Zolotova, N.V. and Ponyavin, D.I., Is the new Grand minimum in progress?, *J. Geophys. Res. (Space Phys.),* 2014b, vol. 119, p. 3281.